2020-21 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Craig/Harutyunyan/H. Zhou, *Real Analysis*
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 202 A-B-C (FWS), Labutin/Putinar, *Complex Analysis*
Analytic functions. Complex integration. Cauchy's theorem. Series and product developments. Entire functions. Conformal mappings. Topics in complex analysis.

MATH 206 A (F), Chandrasekaran, *Matrix Analysis & Computation*
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, *Numerical Simulation*
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Yang, *Numerical Solution of Partial Differential Equations - Finite Difference Methods*
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Garcia-Cervera, *Numerical Solution of Partial Differential Equations - Finite Element Methods*
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Jacob/X. Zhao/Morrison, *Modern Algebra*
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Cooper, *Foundations of Topology*
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), Long, *Homotopy Theory*
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Bigelow, *Differential Topology*
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

~~**MATH 225 A (F), Agboola, *Algebraic Number Theory***~~ **COURSE CANCELLED**
This course is intended to be a quarter-long introduction to basic algebraic number theory. It would be helpful (but not essential) to take this course if you are planning to take 225BC in the Winter and Spring.

A list of topics that will be covered includes:

Basic commutative algebra: Noetherian properties, integrality, rings of integers.

More commutative algebra: Dedekind domains, unique factorisation of ideals, localisation.

Norms, traces and discriminants.

Decomposition of prime ideals in an extension field.

Class numbers and units. Finiteness of the class number: Minkowski bounds. Dirichlet's unit theorem. Explicit calculation of units.

Decomposition of prime ideals revisited: the decomposition group and the inertia group associated to a prime ideal. A nice proof of quadratic reciprocity.

The prerequisites for this course are a solid knowledge of the basic first-year graduate courses in algebra and analysis, and a level of mathematical maturity appropriate for an advanced graduate course.

Some references:

"Algebraic Theory of Numbers", by P. Samuel (recently reprinted as a Dover paperback)
"Algebraic Number Theory" by A. Frohlich and M. J. Taylor (CUP).
"Algebraic Number Theory", by S. Lang (Springer).
"Algebraic Number Theory", by J. Neukirch (Springer)

Math 225 BC (WS) – Castella, *The Arithmetic of Elliptic Curves*
Course description: The study of the arithmetic of elliptic curves is an extremely rich area of mathematics, incorporating ideas from complex analysis, algebraic geometry, group and Galois theory, and of course number theory. Many fundamental problems in number theory, such as Fermat's Last Theorem and the congruent number problem, lead naturally to the study of elliptic curves.

The purpose of this course is to explore some of the basic ideas in this area, including the group law on elliptic curves, the structure of this group over various fields, and the Birch and Swinnerton-Dyer conjecture.

Topics to be covered: The course is structured in two halves (winter and spring).

_ Topics in the first half include: definition of the group law on elliptic curves and explicit formulas, the Lutz{Nagell Theorem, the Mordell {Weil Theorem, and elliptic curves over **C**.
_ Topics in the second half include: modular forms on congruence subgroups of SL**2**(**Z**), **L**-functions of elliptic curves and modular forms, Hecke operators, and the Eichler {Shimura construction.

Prerequisites: Math 220ABC. Some background in Algebraic Number Theory, Complex Analysis, and Algebraic Geometry is helpful but not required.

Grading:
Problem sets (biweekly): 50%
Take-home final exam: 50%
Textbooks: Some recommended textbooks are:

_ A. W. Knapp, Elliptic Curves, Princeton University Press (1992). [The course will have a similar selection of topics as this textbook, but our presentation may di_er from it.]
_ J. H. Silverman and J. Tate, Rational Points on Elliptic Curves, (2nd Edition), Springer (2015).

Math 227 A (F), Manin, *Rational Homotopy Theory*
Rational homotopy theory is a major tool for studying simply connected spaces; essentially, it gives a complete (and not too complicated) algebraic description of such spaces if you're willing to disregard finite-order effects. Its many consequences in topology and geometry include:

* All but a few Riemannian manifolds have infinitely many distinct closed geodesics (Vigué-Poirrier and Sullivan).
* The homotopy automorphism group of a compact simply connected space is an arithmetic group (Sullivan).

In this course I will cover Sullivan's model of rational homotopy theory and these and other applications. I will explain any necessary ideas from algebraic topology which are not covered in Math 221 or 232: higher homotopy groups, fibrations, Postnikov systems, the Serre spectral sequence, and obstruction theory.

MATH 227 B (W), Z. Wang, *Advanced Topics in Geometric and Algebraic Topology*
(3+1)-TQFTs and applications

(3+1)-topological quantum field theories (TQFTs) are much harder to construct than (2+1)-TQFTs. We will start with an introduction to some well-known (3+1)-TQFTs such as the Crane-Yetter TQFTs. Then we will discuss applications to the topology of smooth 4-manifolds, topological phases of matter, and topological quantum computing.

MATH 227C (S), Long, *Advanced Topics in Geometric and Algebraic Topology*
Thin groups and superstrong approximation

I'll discuss issues around expander graphs, thin groups and superstrong approximation. See

https://www.ams.org/journals/notices/201906/rnoti-p905.pdf

for what some of these words mean.

MATH 232 A-B (WS), Bigelow/Manin, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 236 A (F), Goodearl, *Homological Algebra*
This is to be the first quarter of introduction to a subject that provides fundamental tools for many areas of algebra, as well as topology, geometry, and even functional analysis. The major theme could be described as "algebraic construction of homology and cohomology theories". To carry this out, we will introduce and study topics such as: Homomorphism and tensor product functors; projective, injective, and flat modules; exact sequences and resolutions; chain complexes and homology; Ext and Tor functors.

Here is a sketch of one strand of the subject. Typically, given a module X and a surjective module homomorphism f : A --> B, not all homomorphisms X --> B factor through f. In fancier jargon, this means that the induced map [g --> fg] from Hom(X,A) --> Hom(X,B) is not always surjective. One fix is to restrict attention to those modules X for which this induced map is always surjective, namely the "projective" modules. An arbitrary module A can always be expressed as a quotient P_0/K_0 where P_0 is a projective module. For finer information, express K_0 in turn as P_1/K_1 with P_1 projective, and so on. This results in a "projective resolution" of A, namely an infinite sequence ... --> P_2 --> P_1 --> P_0 --> A --> 0 of modules and homomorphisms, where each P_i is a projective module and the kernel of each map equals the image of the one to its left. Thus, studying A by means of its projective resolutions is a way to study arbitrary modules by means of projective ones.

Another approach to the non-surjectivity problem for the map Hom(X,A) --> Hom(X,B) is to measure the lack of surjectivity by means of the quotient group, Hom(X,B) modulo the image of Hom(X,A). After some fine-tuning, this approach leads to a sequence of abelian groups labelled Ext^n(X,C) which measure the deviation from surjectivity of maps between Hom-groups for individual situations. For instance, even if X is not projective, it may still happen that a particular induced map Hom(X,A) --> Hom(X,B) is surjective, and the "vanishing condition" Ext^1(X,ker(f)) = 0 is sufficient to ensure this.

Prerequisites: Math 220ABC or consent of instructor.

Prospective students should have some background in modules, and be comfortable working with them, but are not expected to be experts. In particular, relevant concepts such as projective, injective, and flat modules, tensor products, categories and functors will be developed in the course.

Math 236 B (W), X. Zhao, *Homological Algebra*
In the second quarter of the course, we will introduce a categorical approach to work with complexes and cohomology. For an abelian category (e.g. category of modules over a ring, category of sheaves of abelian groups over a topological space, etc), we construct its derived category. Roughly speaking, the derived category consists of complexes of objects in the abelian category, and two complexes having the same cohomology are now considered isomorphic to each other. The general framework of triangulated categories and localizations will be introduced to construct such a category and to study its properties.

The second part of the course introduces derived functor. After showing the construction, we will focus on derived functors most used in algebra and algebraic geometry, and use them to construct and compute several invariants (Hochschild and cyclic homology, higher direct image sheaves, etc). An introduction to spectral sequences with applications towards computing the composition of derived functors will be covered.

Prerequisite: Math 220ABC and 236A, or consent of instructor.

The students should have a solid background in module theory. We will use sheaves on topological spaces as one major example, the basic definitions will be covered in class.

MATH 240 A-B-C (FWS), Dai/Wei/Ye, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A-B (FW), Wei/Dai, *Topics in Differential Geometry*

MATH 241 C (S), Dai, *Topics in Differential Geometry*
Determinants, Analytic Torsions, and Mirror Symmetry

Determinant of Laplacian is defined using zeta function regularization. It enjoys significant applications in Mathematics and Physics. The analytic torsion is a certain combination of determinants of Laplacians introduced by Ray and Singer as the analytic analog of the Reidemeister torsion in Topology. We will start by introducing the necessary background, and then the basic properties leading to important results such as the holomorphic anomaly formula which plays important roles in its application to Mirror Symmetry.

MATH 246 A-B-C (FWS) Ponce/Sideris, *Partial Differential Equations*
First-order nonlinear equations; the Cauchy problem, elements of distribution theory an Sobolev spaces; the heat, wave, and Laplace equations; additional topics such as quasilinear symmetric hyperbolic systems, elliptic regularity theory.

MATH 260 AA (W) Garcia-Cervera, *Mathematical Foundations of Electronic Structure Theory*
Description of the course:
A complete quantum mechanical description of an electronic system requires the solution of the many-body Schrodinger equation. The numerical approximation of this equation, however, is impractical: using a straightforward numerical discretization, the number of degrees of freedom grows exponentially with the number of electrons, and therefore only systems with very few atoms can be considered. As a result, a number of reduced models have appeared in the literature, and are intensely use in Physics, Chemistry, and Materials Science for the study and development of new materials with desirable properties. These problems have been the source of a significant amount
of work in the Mathematical community. This course is an introduction to the study of the Schr¨odinger Hamiltonian, and some of reduced models used in the study of quantum systems, such as Hartree-Fock, Thomas-Fermi, and Kohn-Sham Density-Functional Theory. To do this, we will introduce some topics from operator theory and the calculus of variations. However, the course will be partly conducted as a research seminar, with discussions of current areas of research and open problems.

Tentative list of topics:
1. One Electron Hamiltonians: The Hydrogen Atom.
2. Many-Body Schr¨odinger Hamiltonian: The HVZ Theorem.
3. Periodic Systems and the study of solids: Bloch theory.
4. Perturbation Theory.
5. Exponential decay of eigenfunctions: Localization.
6. Hartree-Fock, Thomas-Fermi, and Density-Functional Theory (DFT).
7. Beyond DFT: Green's functions and Many-Body methods.

Prerequisites: Basic knowledge of Functional Analysis and PDEs (at the level of Math 201 and Math 243).

References: Although some of the material will be extracted from published research articles, we will use some of the following references:
1. Functional Analysis, by Michael Reed and Barry Simon.
2. Analysis of Operators, by Michael Reed and Barry Simon.
3. Quantum Mechanics, by Messiah.
4. Peturbation Theory of Linear Operators, by Tosio Kato.
5. Quantum Theory of Many-Particle Systems, by Alexander L. Fetter and John Dirk Walecka.

MATH 260EE (FWS) Cooper, *Graduate Student Colloquium*
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260H (S) H. Zhou, *Geometric Inverse Problems*
An important inverse problem arose in geophysics in an attempt to determine the inner structure of the Earth, such as the sound speed, from measurements on the surface of travel times of seismic waves, which is called travel time tomography in seismology. Recent geophysical explorations provide some hint that the substructure, especially the inner core, of the Earth exhibits various anisotropic properties, thus generally the sound speed depends on both the locations and the directions. From a mathematical point of view, the sound speed of the Earth is modeled by a Riemannian metric (i.e. locally a positive definite symmetric matrix function), and the travel times by the lengths of unit speed geodesics between boundary points. In this topics course, we will introduce the mathematical analysis of the above travel time tomography problem, and its linearization, the geodesic X-ray transform. The latter problem is concerned with recovering a function or tensor field on a bounded region from its integrals over curved geodesics, which has important applications in medical imaging, geophysics and physics. We will discuss the uniqueness and stability of these problems, reconstruction method and the partial data case, where one only knows the data on part of the boundary. There are no prerequisites, I will review necessary mathematical background when things are needed.

References:
[1] Inverse problems: visibility and invisibility, G. Uhlmann, JEDP 2012.

MATH 260HH (S) Y. Zhang, *Special Topics in Number Theory*
Modular Forms

First Part: We shall prove the equivalence of (at least) six versions of the Modularity Theorem.

Second Part: We shall cover the necessary materials from Galois representation with the aim to give a proof of the Modularity Theorem.

MATH 260J (W) Atzberger, *Special Topics in Machine Learning*
This course will cover mathematical topics relevant to machine learning to establish rigorous foundations and to provide guidelines for practical applications. As a rigorous basis for inference, the course will draw on results from functional analysis, optimization, statistical learning theory, and culminate in the discussion of active current topics. For example, the approximation abilities of deep learning with neural networks, formulations and training of unsupervised methods such as auto-encoders and GANs, and non-neural network approaches such as support vector machines, kernel methods, and probabilistic methods. The special topics will vary from year-to-year with emphasis on different basic and advanced techniques. A central emphasis will be on the development of rigorous mathematical theory and how this can be used to guide the design of machine learning algorithms, perform training, and carryout analysis to evaluate performance. The beginning introductory materials of the course will use the books "Foundations of Machine Learning," by M. Mohri, A. Rostamizadeh, and A. Talwalkar and "The Elements of Statistical Learning: Data Mining, Inference, and Prediction" by Hastie, Tibshirani, Friedman. The special topics part of the course will be based on materials developed by the instructor and from recent papers in the literature.

More details concerning specific topics can be found below.

Sample of Topics:

🎬Introduction
o History, motivations, and recent developments.
o Statistical Learning Theory, PAC-Learnability, Bayesian Inference.
o Concentration Inequalities and Sample Complexity Bounds.
o No-Free-Lunch Theorems.
o Survey of current topics in data sciences and applications.
🎬
Topics in Supervised Learning
o Neural networks and deep learning approaches.
o Support vector machines, kernel methods, and probabilistic approaches.
o Decision trees.
o Graphical models.
🎬
Topics in Unsupervised Learning
o Manifold learning.
o Neural network auto-encoders and other feature extractors.
o Topological data analysis and persistent homology.
🎬
Advanced Special Topics
o Non-linear optimization methods for machine learning, regularization, stochastic optimization.
o Theory of deep architectures (approximation theory / universality results).
o Reinforcement learning methods, stochastic approximation, and applications.
o Dimensionality reduction, high dimensional probability theory, sparse matrix methods.
o Emerging problems and approaches for applications in the sciences and engineering.

Bibliography:
1. Foundations of Machine Learning, by M. Mohri, A. Rostamizadeh, and A. Talwalkar.
2. The Elements of Statistical Learning Data Mining, Inference, and Prediction, T. Hastie, R. Tibshirani, J. Friedman, (2013).
The course also uses materials developed by the instructor and papers in the literature.

MATH 260L (F), Birnir, *Stochastic Differential Equations*
This course will go through the basic theory of Ito and diffusion processes and then apply this theory to the stochastic Navier-Stokes equation describing turbulence. The books used are Oksendal, Stochastic Diff. Eq. and Birnir, the Kolmogorov-Obukhov Theory of Turbulance. Anyone with basic knowledge of ODEs and probability can participate.

MATH 260Q (W), H. Zimmermann, *Introduction to Representation Theory of Groups and Algebras*

We will start by assembling an arsenal of interesting examples of finite dimensional algebras and by introducing an initial installment of basic (not yet specialized) tools to tackle their representations: a bit of background on categories and functors, chain conditions, decomposition theory.

Then we will focus on the representation theory of finite groups (= the representation theory of the corresponding group algebras). The first major goal is to develop the classical character theory in characteristic zero, including Mackey's results on irreducibility of induced representations (I mention his name specifically, since you may want to look up the obituary for him, which appeared in a recent issue of the Notices of the AMS), culminating in two famous theorems, due to E. Artin and Brauer, respectively. The purely algebraic viewpoint will be supplemented by Young's combinatorial approach to characters by way of "Young tableaux", a crucial ingredient in the representation theory of symmetric groups.

In a second part, we will move to more general algebras. The ultimate goal is to understand the basic building blocks of their representations and classify them, a realistic objective only if the considered algebra has "finite (or tame) representation type", meaning that the number of such building blocks is finite (or controlled infinite). The trip will conclude with the conjectures of Brauer and Thrall proposing characterizations of algebras of finite representation type. These conjectures were confirmed, first by Roiter, then by Auslander in a broader setting. The finessed strategy which we will be adopting will expose us to the "modern" methods of the subject.

MATH 501 (F), Garfield, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.