2019-20 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Harutyunyan/Labutin, ***Real Analysis***
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 206 A (F), Chandrasekaran, ***Matrix Analysis & Computation***
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, ***Numerical Simulation***
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Yang, ***Numerical Solution of Partial Differential Equations - Finite Difference Methods***
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Atzberger, ***Numerical Solution of Partial Differential Equations - Finite Element Methods***
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Castella Cabello, ***Modern Algebra***
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Long, ***Foundations of Topology***
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), McCammond, ***Homotopy Theory***
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Long, ***Differential Topology***
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A (F), Zhang, ***Topics in Number Theory***
The prerequisites for this course are a solid knowledge of the basic first-year graduate courses in algebra and analysis, and a level of mathematical maturity appropriate for an advanced graduate course.

MATH 227 B (W), Z. Wang, *Advanced Topics in Geometric and Algebraic Topology*
Volume conjectures and 3D quantum gravity

Volume conjectures come out of the study of pure 3D quantum gravity with negative cosmological constant. There are several different formulations and many verifications for knots and 3-manifolds. I will discuss their physical origin and focus on an approach based on twisted Alexander polynomials. It is known that a certain limit of twisted Alexander polynomials or Reidermeister torsions gives rise to the hyperbolic volume. This beautiful theorem reduces the volume conjecture to the search of relations between twisted Alexander polynomials and quantum invariants.

MATH 227C (S), Cooper, *Advanced Topics in Geometric and Algebraic Topology*
Geometric structures on Manifolds
This course will be about various kinds of geometric structures. The meaning of geometry we will use is due to Klein: that of a group acting on a space. The group is a Lie group and the space is a manifold. Examples are Euclidean, spherical and hyperbolic geometries. Every compact surface has one of these kinds of geometry. Which kind is governed by the Gauss-Bonnet theorem. In dimension three, the Geometrization theorem of Thurston and Perelman asserts that compact 3-manifolds are built out of eight kinds of Thurston geometry. These correspond to eight Lie groups. The geometries mentioned so far are all subgeometries of real projective geometry. The study of projective structures on manifolds is a young area with many open problems and one I have been studying for some time. Some of the topics we might cover include: Lie Groups and Lie algebras Projective geometry and projective structures on surfaces Hyperbolic structures Teichmuller space and some generalizations Limits of geometric structures and transitions between different kinds of geometric structure. Applying nonstandard analysis (infinitesimals and infinitely large numbers) to geometry.

MATH 228 A (F), Akemann, *Functional Analysis*
Prerequisite: *Mathematics 201A-B-C.*
Topics in functional analysis such as operators on Hilbert space, convex analysis, fixed point theorems, distribution theory, unbounded operators.

MATH 231 A-B (WS), Morrison, *Lie Groups and Lie Algebras*
Differentiable manifolds, definition and examples of lie groups, lie group-lie algebra correspondence, nilpotent and solvable lie algebras, classification of semi-simple lie algebras over the complexes, representations of lie groups and lie algebras, special topics.

MATH 232 A-B (FW), Bigelow/Cooper, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 240 A-B-C (FWS), Dai/X. Zhou/Wei, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A-B (FW), Wei/Dai, *Topics in Differential Geometry*
Comparison Geometry
In Riemannian geometry, by comparing the the right model, one can often obtain optimal estimates, for diameter, volume, eigenvalues in terms of curvature bounds. The subject has applications in solving

many famous conjectures, like Perelman's solution of Poincare conjecture, fundamental gap conjecture. We will discuss various comparison tools and applications.

MATH 241 C (S), Ye, *Topics in Differential Geometry*

MATH 243 A-B-C (FWS) Birnir/H. Zhou, *Partial Differential Equations*
Existence and stability of solutions, Floquet theory, Poincare-Bendixson theorem, invariant manifolds, existence and stability of periodic solutions, Bifurcation theory and normal forms, hyperbolic structure and chaos, Feigenbaum period-doubling cascade, Ruelle-Takens cascade.

MATH 260EE (FWS), Cooper, *Graduate Student Colloquium*
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260H (W) H. Zhou, *Inverse Boundary Value Problems*
In inverse problems one attempts to determine the interior properties of a medium by applying various non-intrusive methods. The mathematical problems under study are often motivated by real world application purposes, including questions arising in medical imaging (e.g. CT, EIT, MRI), geophysics (seismology), mathematical physics, etc.

In this topics course, we expect to introduce the mathematical analysis of inverse boundary value problems (e.g. the inverse conductivity problem), which consists in determining the coefficients (representing the properties of the medium) of a PDE in some bounded domain from the properties of its solutions on the boundary of the domain (e.g. Dirichlet-to-Neumann map). We will discuss the uniqueness and stability of such determination problem, reconstruction method and the partial data case, where one only know the properties of the solutions on part of the boundary.

If time permits, we will also give brief introduction to related inverse problems, such as the anisotropic version of the problem, the linearized problem, invisibility and cloaking. There are no prerequisites, but some basic knowledge of PDEs and Fourier analysis will be helpful for taking the course.

References:
[1] Inverse problems: visibility and invisibility, Gunther Uhlmann, JEDP 2012.

MATH 260J (W) Atzberger, *Special Topics in Machine Learning*
This special topics course will survey select mathematical topics in machine learning, their rigorous foundations, and practical applications. The special topics will vary from year-to-year with emphasis on different basic and advanced techniques. The course will build upon a rigorous basis for inference from statistical learning theory and culminate in the discussion of select special topics such as advanced methods from deep learning based on neural networks, topological data analysis based on persistent homology, or methods for dimension reduction. A central emphasis will be the connection between rigorous mathematical theory and the performance of practical statistical inference algorithms in particular problem domains. Probabilistic approaches for development of learning methods will be emphasized with connections to high-dimensional probability theory and stochastic analysis. The beginning introductory materials of the course will use the books "Foundations of Machine Learning," by M. Mohri, A. Rostamizadeh, and A. Talwalkar and "The Elements of Statistical Learning: Data Mining, Inference, and Prediction" by Hastie, Tibshirani, Friedman. The remaining special topics part of the course will be based on recent papers from the literature and special lecture materials.

More details concerning specific topics can be found below.
Sample of Topics:

- Introduction
  - o Historic developments, motivations, and recent break-throughs.
  - o Statistical Learning Theory, PAC-Learnability
  - o Concentration Inequalities and Sample Complexity Bounds
  - o No-Free-Lunch Theorems
  - o Survey of data sciences and related motivating applications
- Special Topics in Supervised Learning
  - o Neural networks and deep learning methods
  - o Support Vector Machines and modern classification methods
  - o Decision trees
  - o Graphical models
- Special Topics in Unsupervised Learning
  - o Manifold learning
  - o Topological Data Analysis and Persistent Homology
  - o Neural Networks and feature extraction
- Advanced Special Topics
  - o Non-linear optimization methods for machine learning
  - o Theory for design of deep architectures
  - o Regularization and stochastic gradient descent
  - o Sparse matrix methods
  - o Dimensionality reduction
  - o Markov-chain Monte-Carlo sampling for posterior distributions

Bibliography:
1. Foundations of Machine Learning, by M. Mohri, A. Rostamizadeh, and A. Talwalkar.
2. The Elements of Statistical Learning Data Mining, Inference, and Prediction, T. Hastie, R. Tibshirani, J. Friedman, (2013).

MATH 260L (S), Craig, *Optimal Transport in PDE, Geometry, and Applied Mathematics*
Over the past twenty years, optimal transport has emerged as a powerful tool in partial differential equations, geometry, and applied mathematics. While many of these developments are relatively recent, the foundational question of optimal transport is quite old, originally posed by Gaspard Monge in 1781: how can one rearrange a pile of dirt to look like another pile of dirt, using the least amount of effort?

Monge's optimal rearrangements, known as *optimal transport plans,* naturally induce new ways of interpolating between functions, shedding light on convexity properties of energy functionals arising throughout mathematical physics. In partial differential equations, these convexity properties have led to optimal estimates for stability of solutions and asymptotic behavior. In geometry, these convexity results culminated in a synthetic characterization of Ricci curvature, independent of a manifold's underlying differential structure. In applied mathematics, these results led to a range of new numerical methods for simulating solutions of partial differential equations and computing related optimization problems.

In this topics course, we will introduce the foundations of optimal transport and gradient flows on metric spaces, and we will discuss applications of these tools in partial differential equations, geometry, and applied mathematics. We will close by considering recent extensions of the optimal transport framework to discrete spaces, which has led to natural formulations of partial differential equations on graphs, discrete notions of Ricci curvature, and new numerical methods.

References:

[1] *Gradient Flows in Metric Spaces and the Space of Probability Measures*, Ambrosio, Gigli, and Savaré
[2] *Entropic Ricci Curvature for Discrete Spaces*, Jan Maas
[3] *Topics in Optimal Transportation*, Cedric Villani
[4] *Optimal Transport, Old and New*, Cedric Villani

MATH 260Q (FW), Goodearl, *Quantum Groups*
This course will offer an introduction to core ideas in a subject called, somewhat mysteriously, "Quantum Groups". It is a field which arose from mathematical physics in the 1980s and has since developed many connections with areas as diverse as representation theory, noncommutative geometry, and knot theory. Its historical origin was the study of the "quantum Yang-Baxter equation" in quantum statistical mechanics, solutions to which came from the representation theory of "quantized enveloping algebras" of Lie algebras and led to "quantized algebras of functions" on Lie groups.

The prerequisite is just 220ABC or equivalent. Background in Lie theory, representation theory, or algebraic geometry is not assumed, but will be developed as needed, along with some relevant noncommutative algebra. A sample list of topics to be discussed follows.

-- Affine algebraic varieties, polynomial function algebras, and their quantizations
-- Lie algebras, enveloping algebras, and their quantizations
-- The quantum Yang-Baxter equation and R-matrices
-- Noncommutative noetherian rings and rings of fractions
-- Skew polynomial rings and presentations of quantized algebras
-- Hopf algebras
-- Hopf duality between (quantized) algebras of functions on Lie groups and (quantized) enveloping algebras of their Lie algebras

MATH 501 (F), Garfield, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.