2025-2026 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Craig/H. Zhou, ***Real Analysis***
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 206 A (F), Chandrasekaran, ***Matrix Analysis & Computation***
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, ***Numerical Simulation***
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), X. Yang, ***Numerical Solution of Partial Differential Equations - Finite Difference Methods***
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), Atzberger, ***Numerical Solution of Partial Differential Equations - Finite Element Methods***
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Yao/Krishna/Castella, ***Modern Algebra***
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Bigelow, ***Foundations of Topology***
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), MCammond, ***Homotopy Theory***
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Wink, ***Differential Topology***
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A-B-C (FWS), Agboola, ***Algebraic Number Theory***
A three-quarter sequence covering certain fundamental topics in number theory and arithmetic geometry. We shall focus on algebraic number theory, the theory of elliptic curves, and the theory of modular forms.

Math 227A (F), McCammond, *Advanced Topics in Geometric and Algebraic Topology*
Exceptional Mathematics
Many parts of mathematics have classification theorems which state that an object with a particular structure is part of an infinite family or it one of a finite number of exceptions. As one shifts from one topic to another, the infinite families in one area are often connected to the infinite families in other areas, and are part of what one might call "classical mathematics". Similarly, the exceptions in one area are often connected to the exceptions in other areas, and are part of what one might call "exceptional mathematics". In this course I will survey a number of classification theorems, ranging from regular polytopes and Coxeter groups to Lie groups and Lie algebras, regular graphs, sphere packings and finally classification of the finite simple groups, with a particular focus on the exceptional objects in each case.

MATH 227 B (W), Z. Wang, *Advanced Topics in Geometric and Algebraic Topology*
Homology TQFTs and applications
Homology with coefficient Z_p can be turned into topological quantum field theory (TQFT), in particular p=2. We will start with an introduction to the basic theory of TQFTs, then focus on homology TQFTs with Z_2 coefficient in higher dimensions. If time permits, we will also cover their applications to quantum computing: topologization of stabilizer quantum error correction formalism using Z_2 homology, and unification of the stabilizer formalism with topological quantum computing.

MATH 227C (S), Bigelow, *Advanced Topics in Geometric and Algebraic Topology*
Skein theory for planar algebras
Suppose you are interested in representations of a group, or quantum group, and maps between tensor products of representations. In other words, you are interested in the tensor category of its representations. If you are lucky, this can be packaged into a planar algebra, where morphisms are a certain kind of diagrams in the plane, composition is given by stacking vertically, and tensor products are given by stacking horizontally. Skein theory is the study of ways to manipulate these diagrams by local relations. One example is manipulating knot diagrams by Reidemeister moves.

MATH 231 A-B (FW), X. Zhao
Differentiable manifolds, definition and examples of lie groups, lie group-lie algebra correspondence, nilpotent and solvable lie algebras, classification of semi-simple lie algebras over the complexes, representations of lie groups and lie algebras, special topics.

MATH 237 A-B-C (FWS), X. Zhao/Krishna
Affine/projective varieties, Hilbert's Nullstellensatz, morphisms of varieties, rational maps, dimension, singular/nonsingular points, blowing up of varieties, tangent spaces, divisors, differentials, Riemann-Roch theorem. Special topics may include: elliptic curves, intersection numbers, Bezout's theorem, Max Noether's theorem.

MATH 240 A-B-C (FWS), Dai/Wink/Ye, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A-B (FW), Dai/Ye, *Topics in Differential Geometry*
Contact instructors for detailed course information.

MATH 241 C (S), Wei, *Topics in Differential Geometry*
Curvature and Topology
We will study topological implication of sectional and Ricci curvature lower bounds, especially on fundamental groups.

MATH 243 A-B-C (FWS) Harutyunyan, *Ordinary Differential Equations*
Existence and stability of solutions, Floquet theory, Poincare-Bendixson theorem, invariant manifolds, existence and stability of periodic solutions, Bifurcation theory and normal forms, hyperbolic structure and chaos, Feigenbaum period-doubling cascade, Ruelle-Takens cascade.

MATH 260AA (S) Birnir, *Water waves and two-dimensional Euler Equation*
The KdV equation is the leading order approximations to water waves in shallow water. We explain how the real KdV equation describes the surfaces of the waves and the complex KdV equation describes the flow of water beneath the surfaces. The streamlines of the latter equation explain why surfers can duck dive under the waves. Since the KdV equations (real and complex) are completely integrable, this raises the question: Is the water wave problem or equivalently the two-dimensional Euler Equations integrable? We answer this question and explore the ramifications both for smooth and singular solutions.

MATH 260EE (FWS) ???, *Graduate Student Colloquium*
Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260F (W) Jacobs, *Gradient flows*
Given a metric space and an energy functional defined over the space, a gradient flow is a time evolution equation that follows the steepest descent path for the energy. These equations form an important class of PDEs that encompass a wide variety of phenomena such as heat flow, fluid flowing through a porous media, tumor growth, motion by mean curvature, and many more. Furthermore, gradient flows also play a fundamental role in optimization, as virtually all optimization algorithms are built from different discretizations of gradient flows. In this class, we will consider both the theoretical analysis of gradient flows as well as computational algorithms to approximate gradient flows. An emphasis will be placed on ideas that we will develop from convex analysis and convex duality.

MATH 260HH (W) Atzberger, *Topics in Machine Learning: Foundations and Applications*
The ability of recently engineered machine learning algorithms and natural systems to perform inference and generalize well from limited finite data observations poses interesting challenges and open problems for mathematical analysis and for further algorithmic developments. This course covers both related rigorous mathematical foundations of machine learning and also guidelines for developing practical algorithms for applications. As a rigorous basis for inference, the course will draw on results from functional analysis, optimization, convex analysis, statistical learning theory, and culminate in the discussion of active current topics. For example, the approximation and generative abilities of deep learning with recent types of neural networks, formulations and training of unsupervised methods such as autoencoders, VAEs, GANs, and non-neural network approaches such as support vector machines, kernel methods, and probabilistic methods. A central emphasis will be on the development of rigorous mathematical theory and how this can be used to guide the design of machine learning algorithms, perform training, and carry out analysis to evaluate performance. The beginning introductory materials of the course will use the books "*Foundations of Machine Learning*," by M. Mohri, A. Rostamizadeh.

The special topics part of the course will be based on materials in deep learning literature and other topics developed by the instructor and from other recent papers in the literature.

More details concerning specific topics can be found below:
Sample of Topics:
Introduction
O History, motivations, and recent developments.
O Statistical Learning Theory, PAC-Learnability, Bayesian Inference, Variational Inference.
O Concentration Inequalities, Sample Complexity Bounds , No-Free-Lunch Theorems.

Topics in Supervised Learning
O Neural networks and deep learning approaches.
O Support vector machines, kernel methods, and probabilistic approaches.
O Parametric and non-parametric regression and sampling complexity.
O Decision trees, graphical models, tasks for stochastic dynamical systems.

Topics in Unsupervised Learning
O Manifold learning and other inductive biases for reductions.
O Neural network auto-encoders and related feature extractors.
O Generative methods and analysis using tools of measure theory / probability.

Advanced Special Topics
O Scaling limits of deep neural networks and feature learning: Neural Tangent Kernels (NTK), Feature Machines.
O Deep architectures and theory: Transformers, Variational Autoencoders (VAEs), Mamba
O Generative methods: Generative Adversarial Networks (GANs), Diffusion Methods, Score Matching, Contrastive Learning.
O Dimensionality reduction, sparse matrix methods.
O Emerging problems and approaches for applications in the sciences and engineering.

Bibliography:

1. *Foundations of Machine Learning*, by M. Mohri, A. Rostamizadeh, and A. Talwalkar.

2. *The Elements of Statistical Learning Data Mining, Inference, and Prediction,* T. Hastie, R. Tibshirani, J. Friedman, (2013).

The course also uses materials developed by the instructor and papers in the recent literature.

MATH 260J (F) H. Zhou, *Introduction to geometric inverse problems*
An important inverse problem arose in geophysics in an attempt to determine the inner structure of the Earth, such as the sound speed, from measurements on the surface of travel times of seismic waves, which is called travel time tomography in seismology. From a mathematical point of view, the sound speed of the Earth is modeled by a Riemannian metric, and the travel times by the lengths of unit speed geodesics between boundary points. In this topics course, we will introduce the mathematical analysis of the above travel time tomography problem, and its linearization, the geodesic X-ray transform. The latter problem is concerned with recovering a function or tensor field on a bounded region from its integrals over curved geodesics, which has important applications in medical imaging, geophysics and physics. We will discuss the uniqueness and stability of these problems, reconstruction method and the partial data case, where one only knows the data on part of the boundary. There are no prerequisites, I will review necessary mathematical background when things are needed.

MATH 260L (W) Garcia-Cervera, *Operator Theory with Applications to Quantum Mechanics*
Most of the course will be dedicated to the spectral theory of bounded and unbounded operators in a Hilbert space. We will focus on concrete examples in the context of Quantum Mechanics. This course will also serve as an introduction to the study of the many-body Schrodinger Hamiltonian, and some of reduced models used in the study of quantum systems, such as Hartree-Fock, Thomas-Fermi, and Density-Functional Theory. We will also introduce some topics from the calculus of variations.

Here is a tentative list of topics:
1. Bounded operators: The Closed Graph theorem, resolvent set, and spectrum.
2. Unbounded operators on a Hilbert space: Self-adjointness and the Kato-Rellich condition.
3. Spectral theory of self-adjoint operators: Spectral measures and resolution of the identity.
4. Exponential decay of eigenfunctions.
5. Location of the essential spectrum and the RAGE theorem.
6. Perturbation theory of the discrete spectrum.
7. Periodic systems: the Bloch-Floquet transform.

To illustrate the main ideas, we will use the following model problems throughout the course:
1. One electron Hamiltonian and the Hydrogen atom.
2. Many-body Hamiltonian and the HVZ theorem.
3. Hartree-Fock, and Thomas-Fermi-von Weiszacker.

The course should be accessible to students with a working knowledge of linear functional analysis (at the level of Math 201), linear partial differential equations, and the theory of Sobolev spaces.

References: Although some of the material will be extracted from published research articles, we will use some of the following references:
1. *Introduction to Spectral Theory With Applications to Schrodinger Operators*, by P.D. Hislop and I.M. Sigal.
2. *Mathematical Methods in Quantum Mechanics With Applications to Schrodinger Operators,* by Gerald Teschl.
3. *Functional Analysis Vol. I,* by Michael Reed and Barry Simon.
4. *Analysis of Operators Vol. IV*, by Michael Reed and Barry Simon.
5. *Perturbation theory for linear operators*, by Tosio Kato.

MATH 260Q (S) Liu, *Elements of Euler systems and p-adic L-functions*
The mysterious links between arithmetic groups (such as unit groups of number fields, Mordell–Weil groups of elliptic curves, and Chow groups of algebraic varieties) and special values of complex *L*-functions are at the core of several central conjectures in number theory (due to Birch–Swinnerton-Dyer, Deligne, Beilinson–Bloch, Bloch–Kato, and others). Euler systems, often referred to as "arithmetic incarnations" of *L*-functions, are one of the most powerful tools available to date for the study of such links. This course will be an introduction to Euler systems, with a focus on classical examples (such as cyclotomic units) and perspectives motivated by the construction of *p*-adic *L*-functions.

MATH 260RR (S) Craig, *Advanced Measure Theory*
This course will cover advanced topics in measure theory beyond what is normally covered in the 201 sequence. Topics include Haudsorff measure, the Riesz Representation Theorem, weak convergence of measures, Prokhorov's theorem, metrics on the space of measures (total variation, Wasserstein, and generalizations), disintegration and conditional probability, the Law of Large Numbers, the Central Limit Theorem, and Wiener Processes.

MATH 501 (F), Garfield, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.