

Midterm Exam
Math 34 B
Summer 2009
Prof. Ye

Your name:
Your perm number:
Your signature:
Your TA's name:

Scores:
1.
2.
3.
4.
Total:

No credit will be given if no work is presented.

20 extra credit points are included. So you can get as much as 120 points, on the scale of 100 points.

1. (30 points) Find the following integrals

$$\int (3x^3 + 2x^2 - 4x + 1)dx, \int_0^2 3^{t+1} dt, \int (e^{3t} + 1)dt, \int_1^4 (f'(x)g(x) + g'(x)f(x))dx,$$

where we know that $f(1) = g(1) = 2, f(4) = 3, g(4) = -1$.

Solution We have

$$\begin{aligned} \int (3x^3 + 2x^2 - 4x + 1)dx &= \frac{3}{4}x^4 + \frac{2}{3}x^3 - 2x^2 + x + C, \\ \int_0^2 3^{t+1} dt &= 3 \int_0^2 3^t dt = 3 \left[\frac{1}{\ln 3} 3^t \right]_0^2 = \frac{3}{\ln 3} (3^2 - 0) = \frac{27}{\ln 3}, \\ \int (e^{3t} + 1)dt &= \frac{1}{3}e^{3t} + t + C, \\ \int_1^4 (f'(x)g(x) + g'(x)f(x))dx &= \int_1^4 (f(x)g(x))' dx = f(4)g(4) - f(1)g(1) \\ &= 3 \cdot (-1) - 2 \cdot 2 = -3 - 4 = -7. \end{aligned} \tag{0.1}$$

2. 1) (25 points) Find the critical points and determine their type (max, min or inflection) for the following function

$$x^3 - 27x - 1. \quad (0.2)$$

2) (5 extra credit points) Find the critical points and determine their type (max, min or inflection) for the following function

$$(x - 1)(x^2 + 2x + 1) = x^3 + x^2 - x - 1. \quad (0.3)$$

Solution 1) Set $f(x) = x^3 - 27x - 1$. Then

$$f'(x) = 3x^2 - 27. \quad (0.4)$$

Set $f'(x) = 0$, i.e. $3x^2 - 27 = 0$ we find $x^2 = 9$. Hence we obtain two critical points $x_1 = 3$ and $x_2 = -1$. Next we have $f''(x) = 6x$. So $f''(3) = 18 > 0$ and $f''(-1) = -6 < 0$. It follows that $x_1 = 3$ is a minimum point, and $x_2 = -1$ is a maximum point.

2) Set $f(x) = (x - 1)(x^2 + 2x + 1)$. By the product rule we have

$$\begin{aligned} f'(x) &= -(x^2 + 2x + 1) + (x - 1)(2x + 2) \\ &= -(x + 1)^2 + 2(x - 1)(x + 1) \\ &= (x + 1)(-(x + 1) + 2(x - 1)) = (x + 1)(x - 3). \end{aligned} \quad (0.5)$$

Hence we obtain two critical points $x_1 = -1$ and $x_2 = 3$. Next we have

$$f''(x) = 1 \cdot (x - 3) + (x + 1) \cdot 1 = 2x - 2 = 2(x - 1). \quad (0.6)$$

Hence $f''(-1) = -4$ and $f''(3) = 4$. It follows that $x_1 = -1$ is a maximum point and $x_2 = 3$ is a minimum point.

3. A city is being flooded. The water level in the city is rising at the rate of $2t + 3$ feet per hour, t hours after the start of the flood. The height of the tallest building in the city is 200 feet.

1) (25 points) At what time will the city be completely under water? Here you don't need to work out the decimal value of your answer. For example, if you get say $\sqrt{1000}$, then just leave it in this form. Hint: First find the height of the water at the time t , then set it to be 200 to find the answer. You should use the formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (0.7)$$

for solutions of the equation $at^2 + bt + c = 0$.

2) (5 extra credit points) 14 hours after the start of the flood, is the city completely under water? Why?

Solution 1) Let $h(t)$ denote the water level at t hours after the start of the flood. Then we have $h'(t) = 2t + 3$. It follows that

$$h(t) - h(0) = \int_0^t (2t + 3)dt = [t^2 + 3t]_0^t = t^2 + 3t. \quad (0.8)$$

Since $h(0) = 0$ we deduce $h(t) = t^2 + 3t$. To find the time t at which the city went completely under water we solve the equation $h(t) = 200$, i.e.

$$t^2 + 3t = 200. \quad (0.9)$$

We write it as $t^2 + 3t - 200 = 0$. Then we find the solutions

$$t = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-200)}}{2} = \frac{-3 \pm \sqrt{809}}{2} \quad (0.10)$$

Since $t > 0$ we choose the solution

$$t = \frac{-3 + \sqrt{809}}{2}. \quad (0.11)$$

(We have $\sqrt{809} > \sqrt{9} = 3$.)

2) We have

$$h(14) = 14^2 + 3 \cdot 14 = 196 + 42 = 238. \quad (0.12)$$

Hence the city is completely under water at $t = 14$.

4. (30 points) 1) Use the equation of the tangent line to find the linear approximation of $\sin(\pi + 0.1)$. Hint: $\cos \pi = -1$.

2) Find a linear function that agrees with the function $y = \sqrt{x}$ at $x = 9$ and $x = 16$. Use this linear function to approximate $\sqrt{9.1}$.

3) Find the second order approximation of $e^{0.1}$

Solution 1) The equation of the tangent line for the function $y = f(x) = \sin x$ at $x_0 = \pi$ is given by

$$y = f'(\pi)(x - \pi) + f(\pi) = (\cos \pi)(x - \pi) + \sin \pi = -(x - \pi) + 0 = -(x - \pi). \quad (0.13)$$

Hence we obtain

$$\sin x \approx -(x - \pi). \quad (0.14)$$

In particular,

$$\sin(\pi + 0.1) \approx -(\pi + 0.1 - \pi) = -0.1. \quad (0.15)$$

2) We have $\sqrt{9} = 3$ and $\sqrt{16} = 4$. The desired linear function is given by

$$y = \frac{4 - 3}{16 - 9}(x - 9) + 3 = \frac{1}{7}(x - 9) + 3. \quad (0.16)$$

Then we obtain

$$\sqrt{9.1} \approx \frac{1}{7}(9.1 - 9) + 3 = \frac{1}{7} \cdot 0.1 + 3 \approx 3.014. \quad (0.17)$$

3) We have

$$e^{0.1} \approx 1 + 0.1 + \frac{1}{2}0.1^2 = 1.105. \quad (0.18)$$