

## A Survey of Modern Geometry - Spring 2011 Math Colloquium

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Time: Monday 4:00pm-5:20pm  
Old Little Theater 164B

The subject of geometry is considered to be one of the most intuitive areas of mathematics and questions related to shape and distance have engaged thinkers since the earliest civilizations. Euclid's "Elements" gave a definitive treatment of classical geometry over 2000 years ago, but for many that is where the story ends. Geometric thought has a rich history which is often at the forefront of modern science, engineering, and current mathematical research. This course is a seminar intended to engage students with a limited background in formal mathematics as well as aspiring mathematicians by asking fundamental questions about geometric spaces and the important properties which define and characterize them. We will address various answers to these questions which have been studied throughout history. Additionally, we will explore the intuitive and counter-intuitive consequences of geometric ideas and their applications.

Grading: A maximum of two units may be earned in this course. One unit will be awarded for consistent attendance, and one additional unit will be awarded for work on the homework assignments.  
Homework: Each week a short problem set will be assigned, consisting from one to three problems of various types. Each set is expected to be completed and turned in at the beginning of the next class.

Course Outline: This outline is subject to change as dictated by discussions and the interests of the class.  
Week 0: Introduction. Some historical stories, philosophy of mathematics and asking "What is geometry?"  
Week 1: Euclidean geometry. Revisiting the postulates and understanding shapes with rigid transformations.  
Week 2: Foundations of topology. Geometry without distance, basic examples and homeomorphism.  
Week 3: Topology of curves. Connectedness, circles, cut points and the wedge product.  
Week 4: Topology of surfaces. Orientation, compactness, the cartesian product and gluing.  
Week 5: Triangulation of surfaces. The concept of a graph and Euler's formula.  
Week 6: Genus and the classification of closed surfaces.  
Week 7: Curves on surfaces and the fundamental group. Homotopy equivalence of curves.  
Week 8: Introduction to topological manifolds. Atlases and local properties.  
Week 9: Reintroducing distance. General metrics in Euclidean spaces and measuring distances on surfaces.

Texts: These textbooks are not required but contain much of the material which will be covered in this course. Copies will be available on reserve in the Library. Anyone without much experience in mathematics should seriously consider purchasing a copy of "A Very Short Introduction".

A Very Short Introduction to Mathematics, Grower  
Basic Topology, Armstrong  
Geometry and Topology, Stahl