Math 240A Fall 2009 Homework Assignments

Home Work # 7 due Friday, Dec. 4

Page 134 #4;
Page 152 #2, 6;

Home Work # 6 due Friday, November 20

Page 112 #3, 6;
Page 118 #3, 5, 10;
Page 126 #2,6;

Home Work # 5 due Friday, Oct. 30
Page 74 #1,2,3;
Page 79 #1,7;

Home Work # 4 due Friday, October 23

#1. Show that the equation $z = xy$ defines a submanifold in $\mathbb{R}^3$, but its intersection with the xy-plane is not.

#2. Let A be open in $\mathbb{R}^k$, $f : A \to \mathbb{R}$ a differentiable function. Show that the graph of f is a k-manifold in $\mathbb{R}^{k+1}$.

Page 59 #1,2;

Home Work # 3 due Friday, October 16
#1. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a submersion, show that locally f is the canonical projection after a coordinate change, i.e. there is diffeomorphism $g : U \subset \mathbb{R}^n \to V \subset \mathbb{R}^n$ such that locally $f \circ g^{-1}(x^1, \cdots, x^n) = (x^1, \cdots, x^m)$.

#2. Page 10, #1 (there is a typo, it should be $-\epsilon < x < 0$ instead of $-\epsilon \leq x < 0$.)

#3. Let $O(3)$ denote the set of all orthogonal 3 by 3 matrices, considered as a subspace of $\mathbb{R}^9$.
a) Define a C^∞ function $f : \mathbb{R}^9 \to \mathbb{R}^6$ such that $O(3)$ is the solution set of the equation $f(x) = 0$.
b) Show that $O(3)$ is a compact 3-manifold in $\mathbb{R}^9$.

Home Work # 1 due Friday, Oct. 9
Read Chapter I, Page 1-19, three times, once every week

Home Work # 2 due Friday, October 9

#1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Show that all directional derivative of f exist at $(0,0)$, but f is not continuous at $(0,0)$.

(Note given $u = \begin{pmatrix} h \\ k \end{pmatrix}$, any direction in $\mathbb{R}^2$, the directional derivative of f at a along u, $\frac{\partial f}{\partial u}(a)$, can be defined as

$$\frac{\partial f}{\partial u}(a) = \lim_{t \to 0} \frac{f(a+tu) - f(a)}{t}.$$

#2. Page 46, # 7

#3. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be given by the equation $f(x) = \|x\|^2 \cdot x$. Show that f is smooth and that f carries the unit ball $B(0;1)$ onto itself in a one-to one fashion. Show, however, that the inverse function is not differential at 0.

#4. (The polar coordinates) The polar coordinate in $\mathbb{R}^2$ is related to the rectangular coordinates as follows:

$$x = r\cos\theta, \quad y = r\sin\theta.$$

It can also be thought of as defining a mapping $\mathbb{R}^2 \to \mathbb{R}^2$:

$$P: \ (r,\theta) \to (x,y).$$

a) At which point, will P be a local diffeomorphism?

b) Find a maximal domain in which P is a diffeomorphism onto its image.