Homework 4 Problem 9 Solution

Written by TL

The cost of flying a passenger plane consists of fuel cost and hourly pay for the flight crew. The faster a plane flies, the more fuel it uses to fly each mile. Suppose that the cost of fuel to fly one mile is proportional to the speed of the plane. The plane flies a distance of D miles.

(a) Express the time T taken to fly D miles in terms of the velocity V of the plane.

The question is asking us to solve for T. Since

$$Distance = Rate \times Time$$

and our velocity V is rate,

$$T = \frac{D}{V}$$

(b) Express the cost of the fuel F in terms of the velocity V and the distance D. Use K as your proportionality constant.

We are given that the cost to fly 1 mile is proportional to speed:

$$\text{Cost of 1 mile} = (\text{some constant}) \times \text{V}$$

Let K be this constant. Then $K \times V$ is the cost per mile. Since we are flying for D miles, our fuel cost is

$$K \times V \times D$$

(c) Express the cost of the flight crew f in terms of the velocity V, the distance D, and the hourly pay P.

The flight crew is paid hourly. It receives P dollars an hour. Then the cost to hire the flight crew is

$$P \text{ dollars per hour} \times \text{total hours flown}$$

The number of hours flown is just the time we found in (a). Then our crew cost is

$$P \times T = P \times \frac{D}{V}$$

(d) Express the total operating costs C in terms of the velocity V, the distance D, and the hourly pay P.

The total cost will be the cost of the fuel combined with the cost of the crew. We found both of these in (b) and (c). So then cost is the sum of (b) and (c).

$$C = KVD + P\frac{D}{V}$$

(e) If the flight crew costs 200 dollars per hour and the fuel costs 100 dollars per hour when the speed is 300mph how much would it cost to fly 1500 miles at 750 mph.

Let's split this up into first finding the cost of the flight crew and later finding the cost of fuel.

$$P = \text{cost of crew per hour} = 200 \text{ per hour}$$

and since all the information we need is highlighted in green, we can simply input the values into (c). Then the cost of the flight crew is

$$P\frac{D}{V} = 200 \text{ per hour} \times \frac{1500 \text{ miles}}{750 \text{ mph}} = \mathbf{400}$$

Now let us find the cost of fuel, which is KVD. Then to find the fuel cost, we first need to know what K is. To do this, we will use the information given in blue: fuel costs 100 per hour when the speed $V = 300$mph. Remember that $D = VT$, so we can change our cost formula as follows:

$$\text{Cost of fuel} = KVD = KV(VT) = KV^2T$$

Then as we have information for cost per hour, we take $T = 1$ hour and we can now find K:

$$100 \text{ cost per hour} = K(300\text{mph})^2(1 \text{ hour})$$

$$K = \frac{100}{300^2}$$

So then when $D = 1500$ miles and $V = 750$ mph, our cost of fuel is

$$KVD = \frac{100}{300^2} \times (750\text{mph}) \times (1500 \text{ miles}) = \mathbf{1250}$$

Then our total cost is the sum of the two bolded answers:

$$400 + 1250 = \boxed{1650}$$