1. (10 points) Determine each limit.

(a) $\lim\limits_{x\to 0+} e^{-\frac{2}{x}}$

(b) $\lim\limits_{t\to 0} \frac{1}{t}\left(\frac{1}{\sqrt{1-t}} - 1\right)$

(a) let $t = \frac{1}{x}$. Then $t \to \infty$ as $x \to 0^+$.

$$\lim_{x\to 0^+} e^{-\frac{2}{x}} = \lim_{t\to\infty} e^{-2t} = 0.$$

(b). $$\lim_{t\to 0} \frac{1}{t}\cdot\frac{1-\sqrt{1-t}}{\sqrt{1-t}} = \lim_{t\to 0} \frac{1-\sqrt{1-t}}{t\sqrt{1-t}}\cdot\frac{1+\sqrt{1-t}}{1+\sqrt{1-t}}$$

$$= \lim_{t\to 0} \frac{1-(1-t)}{t\sqrt{1-t}\cdot(1+\sqrt{1-t})} = \lim_{t\to 0} \frac{t}{t\sqrt{1-t}(1+\sqrt{1-t})}$$

$$= \lim_{t\to 0} \frac{1}{\sqrt{1-t}(1+\sqrt{1-t})} = \frac{1}{2}.$$

(a)	0
(b)	$\frac{1}{2}$

2. (10 points) Determine the values of a and b that make the given function continuous in the domain of f.

$$f(x) = \begin{cases} x-1 & \text{if } x \le 1-e \\ \ln(1-x)+a & \text{if } 1-e < x < 0 \\ b\cos^{-1}(\frac{x}{100}) & \text{if } 0 \le x \le 100 \end{cases}$$

$f(1-e) = 1-e-1 = -e.$

$\lim_{x\to(1-e)^+} f(x) = \lim_{x\to(1-e)^+} [\ln(1-x)+a] = \ln(1-(1-e))+a$

$= 1+a.$

$\Rightarrow 1+a = -e \Rightarrow a = -1-e.$

$f(0) = b\cdot\cos^{-1}(0) = b\cdot\frac{\pi}{2}$

$\lim_{x\to0^-} f(x) = \lim_{x\to0^-}[\ln(1-x)+a] = \ln 1 + (-1-e)$

$= -1-e.$

$\Rightarrow b\cdot\frac{\pi}{2} = -1-e.$

$\Rightarrow b = \frac{2}{\pi}(-1-e)$

$a = -1-e$

$b = \frac{2}{\pi}(-1-e)$

3. (10 points) Find the horizontal and vertical asymptotes of $y = \dfrac{3x^2 - 1}{x^2 - 5x - 6}$. Justify your answer.

$\boxed{VA}$ $x^2 - 5x - 6 = 0$; $(x+1)(x-6) = 0$; $x = -1, 6$

$$\lim_{x \to -1^-} \frac{3x^2-1}{x^2-5x-6} = \lim_{x \to -1^-} \frac{3x^2-1 \;\oplus}{\underset{\ominus}{(x+1)}\underset{\ominus}{(x-6)}} = +\infty. \Rightarrow x = -1. \text{ is a V.A.}$$

$$\lim_{x \to 6^-} \frac{3x^2-1 \;\oplus}{\underset{\oplus}{(x+1)}\underset{\ominus}{(x-6)}} = -\infty \Rightarrow x = 6 \text{ is a V.A.}$$

$\boxed{HA}$ $$\lim_{x \to \pm\infty} \frac{3x^2-1}{x^2-5x-6} = \lim_{x \to \pm\infty} \frac{3 - \frac{1}{x^2}}{1 - \frac{5}{x} - \frac{6}{x^2}} = 3$$

$\Rightarrow$ $y = 3$.

horizontal asymptote(s) $x = -1$, $x = 6$
vertical asymptote(s) $y = 3$.

4. (10 points) Find the derivatives of the function $g(t) = \sqrt{3t-1}$ using the definition of derivative. State the domain of the function and the domain of its derivative.

$$g'(t) = \lim_{h\to 0} \frac{g(t+h) - g(t)}{h} = \lim_{h\to 0} \frac{\sqrt{3(t+h)-1} - \sqrt{3t-1}}{h}$$

$$= \lim_{h\to 0} \frac{3t+3h-1 - (3t-1)}{h\cdot(\sqrt{3t+3h-1} + \sqrt{3t-1})}$$

$$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}(\sqrt{3t+3h-1} + \sqrt{3t-1})}$$

$$= \frac{3}{\sqrt{3t-1} + \sqrt{3t-1}} = \frac{3}{2\sqrt{3t-1}},$$

Domain of g; $3t-1 \geq 0 \Rightarrow t \geq \frac{1}{3}$.

Doman of g'; $3t-1 \geq 0$ & $3t-1 \neq 0 \Rightarrow t > \frac{1}{3}$

$$\boxed{g'(t) = \frac{3}{2\sqrt{3t-1}}, \quad \mathrm{Dom}(g) = [\tfrac{1}{3}, \infty), \quad \mathrm{Dom}(g') = (\tfrac{1}{3}, \infty)}$$

5. (10 points) Find the first and the second derivatives of

$$f(x) = -x^5 + 2x\sqrt{x} + xe^x - \frac{3}{x} + 4\frac{1}{\sqrt{x}} + \pi x.$$

$$f(x) = -x^5 + 2x^{\frac{3}{2}} + xe^x - 3x^{-1} + 4x^{-\frac{1}{2}} + \pi x$$

$$f'(x) = -5x^4 + 2\cdot\frac{3}{2}x^{\frac{1}{2}} + e^x + xe^x - 3\cdot(-x^{-2})$$
$$+ 4\cdot(-\tfrac{1}{2})x^{-\frac{3}{2}} + \pi$$
$$= -5x^4 + 3x^{\frac{1}{2}} + (1+x)e^x + 3x^{-2} - 2x^{-\frac{3}{2}} + \pi$$

$$f''(x) = -20x^3 + \frac{3}{2}x^{-\frac{1}{2}} + e^x + (1+x)e^x - 3\cdot 2x^{-3}$$
$$- 2\cdot(-\tfrac{3}{2})x^{-\frac{5}{2}}$$
$$= -20x^3 + \frac{3}{2}x^{-\frac{1}{2}} + (2+x)e^x - 6x^{-3} + 3x^{-\frac{5}{2}}$$

$f'(x) = -5x^4 + 3\sqrt{x} + (1+x)e^x + \frac{3}{x^2} - \frac{2}{x\sqrt{x}} + \pi$

$f''(x) = -20x^3 + \frac{3}{2\sqrt{x}} + (2+x)e^x - \frac{6}{x^3} + \frac{3}{x^2\sqrt{x}}$

6. (10 points) Find the tangent line equation to the curve $f(x) = \dfrac{e^x}{x+1}$ at $x = 1$.

$$f'(x) = \frac{e^x \cdot (x+1) - e^x \cdot 1}{(x+1)^2} = \frac{x e^x}{(x+1)^2}$$

$$f'(1) = \frac{e}{2^2} = \frac{e}{4}.$$

$$f(1) = \frac{e}{1+1} = \frac{e}{2}.$$

$$y - \frac{e}{2} = \frac{e}{4} \cdot (x-1).$$

or $y = \frac{e}{4}x - \frac{e}{4} + \frac{e}{2}$

$$\Rightarrow y = \frac{e}{4}x + \frac{e}{4}$$

$$\boxed{y = \frac{e}{4}x + \frac{e}{4}.}$$