

# Calculus with Applications I

Math 3A, Winter 2010

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Answer the following 12 questions. Include units whenever possible. Calculators or books are not allowed. Show all your work for full credit. Correct answers with inconsistent work may not be given credit.

Name: \_\_\_\_\_

Perm number: \_\_\_\_\_

TA's name: \_\_\_\_\_

1. Find the derivatives of the function  $g(x) = (3x - 2)^2$  using the definition of derivative.



2. Find an equation of the tangent line to the curve  $y = x^2e^{-x}$  at the point  $(1, 1/e)$ .

3. Find the derivative of the function.

(a)  $y = \sqrt{\cos 5x}$

(b)  $y = \frac{x}{1 + \ln(3x)}$

(c)  $y = \ln(x\sqrt{x^2 + 3x + 1})$

(a)

(b)

(c)

4. Consider the curve

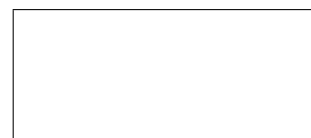
$$e^{\frac{y}{x}} - e = \tan(x - y) + \ln(xy^2)$$

- (a) Find  $dy/dx$  by implicit differentiation.  
(b) Find an equation of the tangent line to the curve at the point  $(1, 1)$ .

(a)

(b)

5. Find the derivative of  $\sin^{-1}(e^{3x})$ .



6. Use the logarithmic differentiation to find the derivative of the function

$$y = \frac{x^x}{\sqrt{x^2 - 1}}$$

7. A cylindrical tank with radius 10 m is being filled with water at a rate of  $3m^3/min$ . How fast is the height of the water increasing?

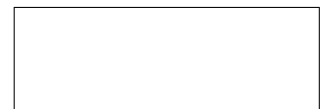
8. Find the absolute maximum and absolute minimum values of  $f$  on the given interval.

$$f(x) = x^2 e^{-2x}, \quad [-2, 4]$$

maximum value

minimum value

9. If  $f(-1) = 2$  and  $f'(x) \leq 5$  for  $-1 \leq x \leq 3$ , how large can  $f(3)$  possibly be? Justify your work.



10. Determine each limit.

(a)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x^2 + 5x}$

(b)  $\lim_{x \rightarrow \infty} (x^5 e^{-2x})$

(c)  $\lim_{x \rightarrow 1} (5 - 4x)^{\frac{2}{x-1}}$

(a)

(b)

(c)

11. Sketch the curve. Find the domain,  $x$  and  $y$  intercepts, asymptotes, intervals on which it is increasing and decreasing, local maximum and minimum, intervals on which the function is concave up and down, and all inflection points.

(a) (25 points)  $f(x) = \frac{x}{x^2 - 4}$

(b) (25 points)  $f(x) = \sqrt{x+2} - \frac{x}{4}$

12. A rectangular storage container with an open top is to have a volume of  $10m^3$ . The length of its base is twice the width. Material for the base costs \$ 10 per square meter. Material for the sides costs \$ 6 per square meter. Find the cost of materials for the cheapest such container.

