

Calculus for Social and Life Sciences II

Math 34B, Spring 2008

Instructor: Sookyung Joo

June 9th, 2008

Answer the following 12 questions. Include units whenever possible. Calculators or books are not allowed. You are allowed to use a 3×5 note. Show all your work for full credit. Correct answers with inconsistent work may not be given credit.

Name: _____

Perm number: _____

Lecture time: _____

TA's name: _____

1. Evaluate the following integrals.

(a) $\int (2e^{4x} + x^3 - 1) dx$

(b) $\int_0^\pi \sin(3t) dt$

(a)

(b)

2. Find the derivatives.

(a) $(x^2 - 3x)e^{-x}$

(b) $e^{-2t} \sin(4t)$

(a)

(b)

3. (a) Find the linear approximation to the function $f(x) = e^{0.2x}$ for the range of values $0 < x < 2$. Do this by making the linear approximation equal the function at the end points $x = 0$ and $x = 2$.
- (b) Find the percentage error in this approximation when $x = 1$.

(a)

(b)

-
4. The number of bees in a forest is growing at a rate of $30 + 4t + 10e^{0.1t}$ bees per day, t days after being introduced into the forest.
- (a) Express this as a differential equation.
- (b) If initially 1000 bees we introduced, how many bees are there after 10 days?

(a)

(b)

5. The radioactive isotope X decays so as to satisfy the differential equation $Q' = -0.05Q$, where Q is measured in milligrams and t in years.
- (a) Determine the half-life of X .
 - (b) If 50 mg of X are present today, how much will remain in 10 years?

(a)

(b)

6. (a) Write down the equations of the isoclines of the equation $y' = 4 - y^2$ for each of the slopes $C = -5, 0, 3$.
- (b) Sketch the slope field using the isoclines.
- (c) What do you think the long term behaviors of these solution are?

$$(a)C = -5;$$

$$(a)C = 0;$$

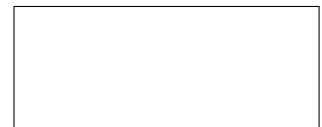
$$(a)C = 3;$$

$$(c)$$

7. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of $200^\circ F$ when freshly poured, and one minute later has cooled to $190^\circ F$ in a room at $70^\circ F$, determine when the coffee reaches a temperature of $150^\circ F$.



8. Find the solution of the equation $g' = 7 - 2g$ with the initial condition $g(0) = 2$.



9. The rabbit population in a park is growing with a maximum capacity of 500. The population is increasing at a rate given by the logistic equation. Initially there was 100 rabbits and the rate of increase was 3 rabbits per day.
- (a) Write down the logistic equation satisfied by the rabbit population.
 - (b) When will the population be 250?
 - (c) What is the population 40 days after the population was 100?

(a)

(b)

(c)

10. Find the partial derivatives with respect to x and y of

(a) $4xy^2 + \frac{x}{y^3}$

(b) $e^{3x-5y} + \sin(xy)$

$$(a)f_x =$$

$$(a)f_y =$$

$$(b)f_x =$$

$$(b)f_y =$$

11. (a) Find the equation of the tangent plane to $z = 3x^2 + 2y^2$ at the point $(x, y) = (1, 3)$.
(b) Apply the tangent plane approximation to find $f(1.01, 3.02)$ where $f(x, y) = 3x^2 + 2y^2$.

(a)

(b)

12. Find the x and y values at the maximum of $f(x, y) = 3x^2 + 5xy + 2y^2 + 4x + 4$.

