Quiz 1.

Note Title 2010-05-03

1. $\begin{cases} x' = y \\ y' = -x + x^3 \end{cases}$ $\qquad f(x,y) = \begin{pmatrix} y \\ -x+x^3 \end{pmatrix}$

(a) This is Hamiltonian;

let $H(x,y) = \frac{1}{2}y^2 + \frac{1}{2}x^2 - \frac{1}{4}x^4$.

Then $x' = H_y$

& $y' = -H_x$.

(b) $y = 0$, $x(x^2-1) = 0$; $x = 0, \pm 1$.

Equilibrium pts are $(0,0)$, $(-1,0)$ & $(1,0)$.

(c) $Df(x,y) = \begin{pmatrix} 0 & 1 \\ -1+3x^2 & 0 \end{pmatrix}$

(i) $Df(\pm 1, 0) = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$

Since it has determinant $-2 < 0$, the linearized system has a saddle, & the equilibrium pts $(\pm 1, 0)$ are hyperbolic.

By Hartman-Grobman, $(\pm 1, 0)$ is saddle & thus unstable.

(ii) $Df(0,0) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow \lambda = \pm i \Rightarrow (0,0)$; non-hyperbolic.

Since $H(x,y)$ has a strict local min at 0,

H can be used as a Liapunov ftn.

i.e. Let $V = H$.

Then $\dot{V}(x(t), y(t)) = \nabla V \cdot \begin{pmatrix} x' \\ y' \end{pmatrix}$

$$= (V_x \quad V_y) \cdot \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

$$= (H_x \quad H_y) \cdot \begin{pmatrix} H_y \\ -H_x \end{pmatrix} = 0$$

$\Rightarrow (0,0)$ is stable, but not asymptotically stable.