

Quiz 1.

Note Title

2010-02-09

Given $A = \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix}$, find e^{At} .

Sol Since A is a triangular matrix, the eigenvalues are 1 & -2.

$$A - I = \begin{pmatrix} 0 & 3 \\ 0 & -3 \end{pmatrix} \Rightarrow v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \text{eigenvector ass. w. } 1$$

$$A + 2I = \begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix} \Rightarrow v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \text{e. vector ass. w. } -2$$

$$\text{Let } P = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\text{Then } A = PDP^{-1}$$

$$P^{-1} = \frac{1}{1 \cdot (-1)} \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$e^{At} = P \cdot e^{Dt} \cdot P^{-1}$$

$$= \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} e^t & 0 \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} //$$