Review sheet for midterm

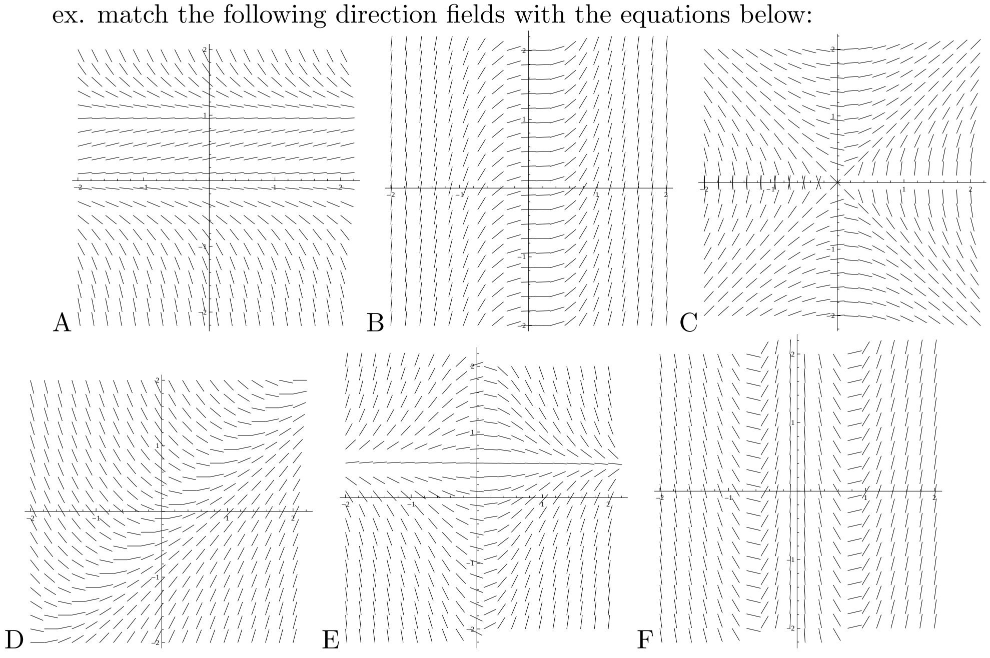
Note the problems here are just meant for extra practice. The assigned homework is a good source for practice, and if you want more practice after these ones, just grabbing problems from the book is a good idea.

1. Direction Fields(slope fields)

You should know what a direction field is. You should know how to create them using isoclines and given a slope field you should know how identify it with it's associated equation. In either case (given a slope field or having to create one) you should be able to draw solution curves. You should be able to identify equilibrium solutions and tell me their stability.

The kind of questions I could ask related to direction fields are to draw solutions curves on a given slope field, To match slope fields to equations, to draw isoclines, or to determine equilibrium solutions and stability. You should also be able to draw a phase line if the equation is autonomous.

ex. match the following direction fields with the equations below:



___ $y' = y(1-y)$

___ $y' = 3t^2$

___ $y' = -2ty + t$

___ $y' = t - y$

___ $y' = -2/t + 4t$

___ $y' = t/y$

ex. Draw the isoclines of the following equations:

$$y' = y^2 - t$$

$$y' = 2t + y$$

ex. Find the equilibrium solutions and their stability of all 8 equations listed here.

2. Separation of Variables

This was the first method we learned of how to explicitly solve differential equations. This problems would probably be explicitly asking to solve a DE where the integrals are difficult.

ex.
$y' = \frac{t(y+1)\cos t}{y}$
$y' = (y^2 - 1)(2t)\cos(t^2)\sin^3(t^2)$
1.3 problems 1-24 are all good example problems

3. Integrating factors/Undetermined Coefficients

This section was our second way to solve differential equations. Many problems would allow you to use either method, although it is possible to create problems where one method would be difficult while the other would be easy, so it is your best interest to know both. But I will not say that you must solve a specific method for a given problem.

2.2 numbers 1-20, 22-30, and 35-40 are good practice

4. Growth and decay

These are problems related to half life, saving accounts, and population growth
all 2.3 problems are good for practice

5. Mixing

These are problems where we have a tank full of brine where there is water flowing in and out of the tank

2.4 numbers 1-12

6. Cooling

Newton's law of cooling, can of soda out of fridge warming up, coffee cooling down...dead bodies

2.4 numbers 15-22

7. Logistic Equation

Population where carrying capacity is an issue. If I ask a problem involving the logistic equation, I will provide the formula

$$y(t) = \frac{L}{1 + \left(\frac{L}{y_0} - 1\right) e^{-rt}}$$

2.5 13-18

ex. You are observing bacteria in a petri dich. Over short times, each bacterium gives rise to 100 bacteria per hour. Over a long time however, the bacterial population levels out at 100,000,000 bacteria. If B is the number of bacteria and t is time in hours, write a differential equation describing the growth of the bacterial population.

ex. Suppose the quantity $y(t)$ exhibits logistic growth. If the values of $y(t)$ at times $t = 0, t = 1, t = 2$ are $3, 5, 6$ respectively, Find the carrying capacity

8. Systems of equations

This is where we have 2 separate differential equations affecting each other. Given one of these problems, you should be able to find the nullclines, find the directions of motion, draw solution curves, identify equilibrium solutions, and tell me whether or not the equilibrium points are stable or unstable. Also, note that these are vector fields, as opposed to the direction fields which we did earlier. You should know the difference between the 2. In class we did an example using a predator prey model, you should be able to interpret a similar model on an exam.

2.6 numbers 1-30 and example 4

9. Trig you need to know

you need to know the value of all 6 trig functions for $0, \pi, \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}$ and multiples of those values.

You need to know how the 6 trig functions relate to one another (like $\tan x = \frac{\sin x}{\cos x}$)

All derivatives of the basic trig functions and arcsin and arctan

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$