MATH CS 120 - FALL 2009

Solve all 12 problem with your group. Write your answers clearly.

Due Nov. 30 in class

1)- Let $a\,,b > 0$. Use complex methods to evaluate

$$\int_{-\infty}^{\infty} \frac{cos(ax)}{x^2+b^2}\,dx.$$

2)- Identify ALL entire functions $f\colon \mathbb{C} \to \mathbb{C}$ such that $|f(z)| \leq |\,z|^3$ for all $z \in \mathbb{C}$.

3)-Determine the image of the region

$$A = \{x + iy \in \mathbb{C} \,:\, 1 < xy < 2, \;\; x > y > 0\},$$

under the transformation $F(z) = z^2$.

4)- Find a harmonic function $u(x, y)$ defined on $\{(x, y) \in \mathbb{R}^2 : x > 0, \;\; y > 0\}$ such that $\lim_{y\downarrow 0} u(x, y) = 1$ for $x > 0$ and $\lim_{x\downarrow 0} u(x, y) = 0$ for $y > 0$.

5)- Let $P_n(z)$ be a complex polynomial of degree n, and define

$$M(r) = \max_{|z|=r} |P_n(z)|, \quad r > 0.$$

Prove that the function $F(r) = \frac{M(r)}{r^n}$ is non-increasing.

6)- Calculate

$$\int_\gamma \ln(z)\,dz,$$

where $\gamma(t) = e^{it}$, $t \in [0, 2\pi]$, and (a) $\ln(1) = 0$, and (b) $\ln(i) = \pi i/2$.

7)- (a) Prove that the function $\sin(1/(1-z))$ has a sequence of zeros converging to the point $z = 1$.
(b) Does this contradict the uniqueness theorem proved in class?

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

8)- Evaluate

$$\int_0^{\infty} \frac{dx}{x^p\,(x+1)}, \quad p \in (0,1).$$

HINT : Define $\ln(z) = \ln|z| + i\,arg(z)$, $\arg(z) \in (0, 2\pi)$. Use this to define z^p, and use the contour described in the picture.

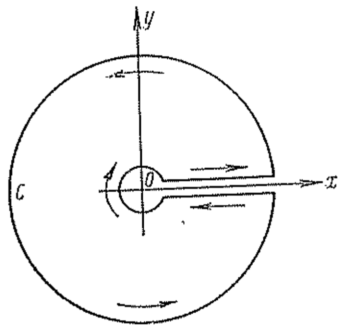

FIG 17

9)- Evaluate

$$\int_0^{\infty} \frac{\ln(x)\,dx}{x^2 + a^2}, \quad a > 0.$$

HINT : Use the contour described in the picture.

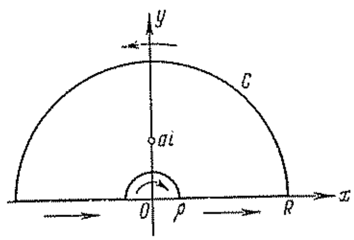

FIG 19

10)- Prove that the series

$$\sum_{k=1}^{\infty} \frac{z^n}{1 + z^{2n}}$$

converges in both the interior ad the exterior of the unit circle and represents an analytic function in each region.

11)- Find the maximum of $|e^{z^2}|$ on the unit disk.

12)- In each case find all complex numbers such that :
(a) $\sin(z) = 0$,
(b) $\cos^2(z) = -1$.