

# Math 240B: Problem Set VI

March 6, 2009

Due: Friday, March 13, 2009.

**Exercise X.** a. Show that in Minkowski space-time  $\mathbb{L}^4$ ,

$$\star\star = (-1)^{k+1} : \Omega^k(\mathbb{L}^4) \longrightarrow \Omega^k(\mathbb{L}^4).$$

b. Determine  $\star dt$ ,  $\star dx$ ,  $\star dy$  and  $\star dz$ .

c. Show that Maxwell's equations can be expressed in terms of the Faraday tensor (described in the lecture notes) as

$$d\mathcal{F} = 0, \quad d(\star\mathcal{F}) = \star(4\pi\mathcal{J}), \quad \text{where } \mathcal{J} = -\rho dt + J_x dx + J_y dy + J_z dz.$$

**Exercise XI.** Suppose that  $M = \mathbb{S}^2$ , the standard unit two-sphere in  $\mathbb{E}^3$ , with Riemannian metric expressed in spherical coordinates as

$$\langle \cdot, \cdot \rangle = (\sin^2 \phi) d\theta \otimes d\theta + d\phi \otimes d\phi.$$

Determine the Laplace operator on functions in this case.