Math 240B: Problem Set II (Revised)

January 30, 2009

Due: Friday, February 6, 2009.

Exercise III. Consider the torus $M^2 = S^1 \times S^1$ with imbedding

$$F : U \to S \quad \text{by} \quad \mathbf{x}(u,v) = \begin{pmatrix} (2+\cos u)\cos v \\ (2+\cos u)\sin v \\ \sin u \end{pmatrix},$$

where u and v are the angular coordinates on the two S^1 factors, with $u+2\pi = u$, $v + 2\pi = v$.

a. Calculate the components g_{ij} of the induced Riemannian metric on M^2.

b. Calculate a continuously varying unit normal $\mathbf{N}$ and the components h_{ij} of the second fundamental form of M^2.

c. Determine the Gaussian curvature K.

Exercise IV. Suppose that $A = (a^i_j)$ and $B = (b^i_j)$ are $n \times n$ real matrices and

$$X_A = \sum_{i,j,k=1}^{n} a^i_j x^k_i \frac{\partial}{\partial x^k_j}, \quad X_B = \sum_{i,j,k=1}^{n} b^i_j x^k_i \frac{\partial}{\partial x^k_j}$$

are the corresponding left invariant vector fields on $GL(n,\mathbb{R})$. Show that

$$[X_A, X_B] = X_{[A,B]}, \quad \text{where} \quad [A,B] = AB - BA.$$