Mathematics 5B Spring 2011: Review for Midterm 2
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Part I. Multiple Choice. There will be several multiple choice questions in which you need to circle the correct answer. Here are some samples:

1. The critical point(s) of the function $f(x,y) = (e^x - 1)(y - 2)$ is/are

a. $(0,2)$ b. $(1,2)$ c. $(0,2)$ and $(1,2)$

d. $(-1,2)$ and $(1,2)$ e. None of these

2. The function $f(x,y) = x^2 + 6xy + 4y^2$ has a critical point at $(0,0)$ which is

a. a local minimum b. a local maximum c. a saddle point

d. degenerate e. None of these

3. The vector field

$$\mathbf{F}(x,y,z) = y\mathbf{i} + ax\mathbf{j} + e^z\mathbf{k}$$

is conservative (that is, it is the gradient of a function) exactly when $a =$

a. -1 b. 0 c. 1 d. π e. None of these

4. The volume enclosed by the ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} = 1,$$

$\frac{4}{3}\pi \cdot 2 \cdot 2 \cdot 3 = 16\pi$

is

a. 12π b. 17π c. 36π d. 144π e. None of these

5. Suppose that the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ is defined by $T(u,v) = (2u + v, u + 3v)$. If

$D^* = \{(u,v) \in \mathbf{R}^2 : 0 \le u \le 1, 0 \le v \le 1\}$, then the area of $T(D^*)$ is

a. 1 b. 3 c. 5 d. 7 e. None of these

6. Suppose that a change of coordinates is given by the transformation

$$T(u,v) = (x(u,v), y(u,v)) = (u^2, u + v).$$

Then within the integrand of a double integral,

$$dxdy = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv, \qquad \text{where} \qquad \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv =$$

a. $dudv$ b. $2dudv$ c. $ududv$ d. $2ududv$ e. None of these

Part II. There will also be a section in which you need to give complete answers to questions. Here are some typical questions:

1. Evaluate the line integral

$$\int_{\mathbf{c}} x^2dx + zdy - ydz,$$

where $\mathbf{c} : [0, 2\pi] \to \mathbb{R}^3$ is the parametrization

$$\mathbf{c}(t) = (t, \sin t, \cos t)$$

of the helix, with the parameter t ranging from 0 to 2π.

2. Find the volume under the paraboloid $z = 4 - x^2 - y^2$ and above the (x, y)-plane.

3. Use spherical coordinates to evaluate the triple integral

$$\int\int\int_W (x^2 + y^2 + z^2)dxdydz,$$

where W is the part of the ball $x^2 + y^2 + z^2 \leq 1$ which lies in the octant $x \geq 0$, $y \geq 0$, $z \geq 0$.

4. Calculate the area bounded by the ellipse

$$(2x)^2 + (x + 3y)^2 = 1,$$

using the following steps:

a. First set $u = 2x$ and $v = x + 3y$, and solve for x and y in terms of u and v.

b. Then construct a linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(u, v) = (x(u, v), y(u, v))$, which takes the unit circle $u^2 + v^2 = 1$ in the (u, v)-plane to the ellipse.

c. Evaluate the integral

$$\int_D 1dxdy,$$

where D is the domain bounded by the ellipse

$$(2x)^2 + (x + 3y)^2 = 1,$$

using the change of variable formula and the fact that the area bounded by the unit circle in the (u, v)-plane is π.