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Mathematics 5B Spring 2011: Review for Midterm 1
April 20, 2011
Professor J. Douglas Moore

Part I. Multiple Choice. There will be several multiple choice questions in which you need to circle the correct answer.

TRIG REVIEW: You should remember the sines and cosines for the following angles:

$$\sin\left(\frac{\pi}{6}\right)=\frac{1}{2},\quad \sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2},\quad \sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}.$$

$$\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2},\quad \cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2},\quad \cos\left(\frac{\pi}{3}\right)=\frac{1}{2}.$$

1. The linear function $F:\mathbb{R}^2\to\mathbb{R}^2$ defined by

$$\begin{pmatrix}x_1\\x_2\end{pmatrix}=F\begin{pmatrix}y_1\\y_2\end{pmatrix}=\begin{pmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix}$$

represents a counterclockwise rotation through the angle

a. $\pi/6$ b. $\pi/4$ c. $\pi/3$ d. $\pi/2$ e. None of these

2. Using the fact that a counterclockwise rotation G through angle ϕ,

$$\begin{pmatrix}y_1\\y_2\end{pmatrix}=G\begin{pmatrix}z_1\\z_2\end{pmatrix}=\begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}z_1\\z_2\end{pmatrix},$$

followed by a counterclockwise rotation F through angle θ,

$$\begin{pmatrix}x_1\\x_2\end{pmatrix}=F\begin{pmatrix}y_1\\y_2\end{pmatrix}=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix},$$

results in a counterclockwise rotation $F\circ G$ through the angle $\phi+\theta$, one finds that $\cos(\phi+\theta)=$

a. $\cos\theta\cos\phi-\sin\theta\sin\phi$ b. $\cos\theta\cos\phi+\sin\theta\sin\phi$ c. $\sin\theta\cos\phi-\cos\theta\sin\phi$

d. $\sin\theta\cos\phi+\cos\theta\sin\phi$ e. None of these

3. From the preceding formula, one finds that $\cos(2\theta) =$

a. $\cos^2\theta - \sin^2\theta$ b. $\cos^2\theta + \sin^2\theta$ c. $\sin\theta\cos\theta$

d. $2\sin\theta\cos\theta$ e. None of these

Note how ideas regarding linear transformations and matrices (material from 3C and 5A) allow you to recover formulae from trigonometry.

The above questions focused on reviewing trigonometric identities. Multiple choice questions on the actual exam will actually be more similar to the questions on the quizzes, or like WebWork questions. Some may involve the chain rule.

Part II. There will also be a section in which you need to give complete answers to questions. Here are some typical questions:

1. Find an equation for the plane in $\mathbb{R}^3$ passing through a given point and perpendicular to a given vector.

2. Find an equation for the plane in $\mathbb{R}^3$ passing through three given points.

3. Find an equation for the plane tangent to the surface $z = x^2 + 2y^2$ at a given point on the surface.

4. Find the length of the curve $y = \cosh x$, when x ranges throughout $[-1, 1]$.

5. Find an equation for the plane tangent to the ellipsoid

$$f(x, y, z) = 17, \qquad \text{where} \qquad f(x, y, z) = x^2 + 4y^2 + 9z^2,$$

at the point $(2, 1, 1)$.

6. If $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ is the function defined by

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \mathbf{F}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1 \sin y_2 + 7 \\ y_1^2 - 3y_2 \end{pmatrix}$$

what is $D\mathbf{F}(3, \pi/3)$?

7. What are the critical points of the function $f(x, y) = (1/2)y^2 - \cos x$? Which of the critical points are local maxima? Which are local minima? Which are saddle points?

8. Find the maxima of the function

$$f(x, y, z) = 2x + y + 3z \quad \text{subject to the constraint} \quad x^2 + 2y^2 + 3z^2 = 7.$$

Problems like 7 and 8 will definitely be on the midterm.