Print Name: Key

Sign Name:

TARDIS CODE:				

Mathematics 5B Spring 2011: Lecture Quiz 6
June 1, 2011
Professor J. Douglas Moore

Multiple Choice. Circle the best answer to each of the following questions. Each question is worth 2 points.

1. Suppose that $\mathbf{S}$ is the boundary of the unit cube

$$0 \le x \le 1, \quad 0 \le y \le 1, \quad 0 \le z \le 1$$

and that $\mathbf{F} = (e^y \sin z, ze^x, z-4)$. If $\mathbf{N}$ is the outward-pointing unit normal to $\mathbf{S}$, then

$\nabla \cdot \vec{F} = 1$

$$\int\int_S \mathbf{F} \cdot \mathbf{N} dA = \int\int_S \mathbf{F} \cdot d\mathbf{S} = \int_0^1 \int_0^1 \int_0^1 1 \, dx\, dy\, dz =$$

a. 1   b. 2   c. 3   d. 4   e. None of these

2. Suppose that $\mathbf{S}$ is the part of the plane $x + y + z = 1$ which lies in the octant defined by $x \ge 0$, $y \ge 0$ and $z \ge 0$. If $\mathbf{C}$ is the boundary of $\mathbf{S}$ oriented counterclockwise as viewed from above and $\mathbf{F}$ is a vector field on $\mathbb{R}^3$ such that $\nabla \times \mathbf{F} = -\mathbf{k}$, then

$\vec{x}(u,v) = \begin{pmatrix} u \\ v \\ 1-u-v \end{pmatrix}$ $\frac{\partial \vec{x}}{\partial u} \times \frac{\partial \vec{x}}{\partial v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} du\,dv$

$$\int_{\mathbf{C}} \mathbf{F} \cdot \mathbf{T} ds = \int_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{x} = \int_0^1 \int_0^{1-u} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} du\,dv = \ldots =$$

a. -1 b. $-1/2$ c. $1/2$ d. 1   e. None of these

3. Suppose that $(\nabla \times \mathbf{F})(x, y, z) = \mathbf{J}(x, y, z)$ on all of $\mathbb{R}^3$, and that outside the solid cylinder

$$D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \le (1/4)\}, \quad \mathbf{F}(x, y, z) = \left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0 \right).$$

If $\mathbf{S}$ is the unit disk in the (x, y)-plane centered at the origin, oriented with the upward pointing unit normal $\mathbf{N}$, then it follows from Stokes' Theorem that

$$\int\int_{\mathbf{S}} \mathbf{J} \cdot \mathbf{N} dA = \int\int_{\mathbf{S}} \mathbf{J} \cdot d\mathbf{S} = \int_{\partial \vec{S}} \frac{-y\,dx + x\,dy}{x^2+y^2} = \int_0^{2\pi} dt =$$

a. -2π b. $-\pi$ c. π d. 2π e. None of these