Math 3CI: Homework
Linear algebra

November 18, 2009

Write up a clear solution to each of the following problems and turn them in by 5pm on Monday, November 23. You should turn it in to Robert Ream, who has office hours in the Math Lab from 3-5 on Monday, November 23.

I. a. Suppose that V is the space of solutions to the homogeneous linear differential equation

$$\frac{d^2y}{dt^2} - y = 0. \tag{1}$$

It follows from the theory of homogeneous linear differential equations that V is a vector space of functions. Find a basis for V. In other words, find a collection of functions $(f(t), g(t))$ such that the general solution to the differential equation (1) is

$$y(t) = c_1 f(t) + c_2 g(t).$$

b. Suppose that V is the space of solutions to the homogeneous linear differential equation

$$\frac{d^2y}{dt^2} + y = 0.$$

Find a basis for V. (Hint: This is the equation of simple harmonic motion, and you should REMEMBER the general solution.)

c. Suppose that V is the space of solutions to the homogeneous linear differential equation

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = 0.$$

Find a basis for V. (Hint: For elements of the basis, try functions of the form $t^k e^t$.)

d. Suppose that V is the space of solutions to the homogeneous linear differential equation

$$\frac{d^4y}{dt^4} - 2\frac{d^2y}{dt^2} + y = 0. \tag{2}$$

Find a basis for V.

II.a. Write out the coefficient matrix for the homogeneous linear system of equations

$$\begin{array}{rrrrrcl} x_1 & +2x_2 & +x_3 & +2x_4 & +3x_5 & = & 0, \\ x_1 & +2x_2 & +2x_3 & +3x_4 & +4x_5 & = & 0, \\ 2x_1 & +4x_2 & +2x_3 & +4x_4 & +7x_5 & = & 0. \end{array} \tag{3}$$

b. Use the elementary row operations to put the matrix in row-reduced echelon form.

c. Write the general solution to the homogeneous linear system in terms of a basis for the linear subspace V of solutions to (2).

d. What is the dimension of V?