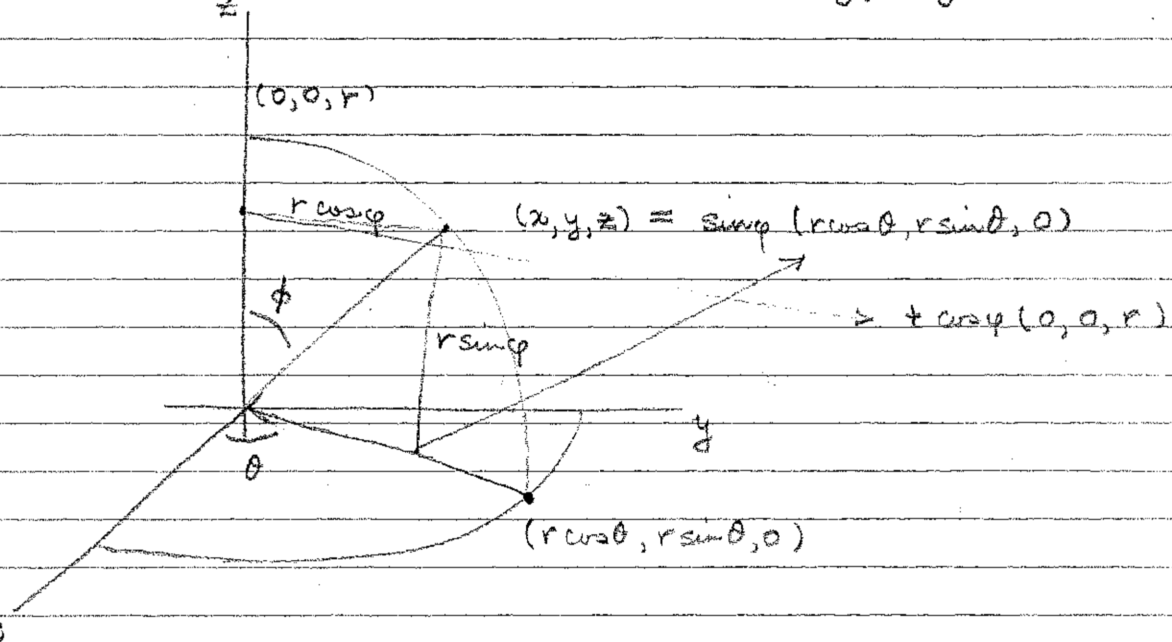
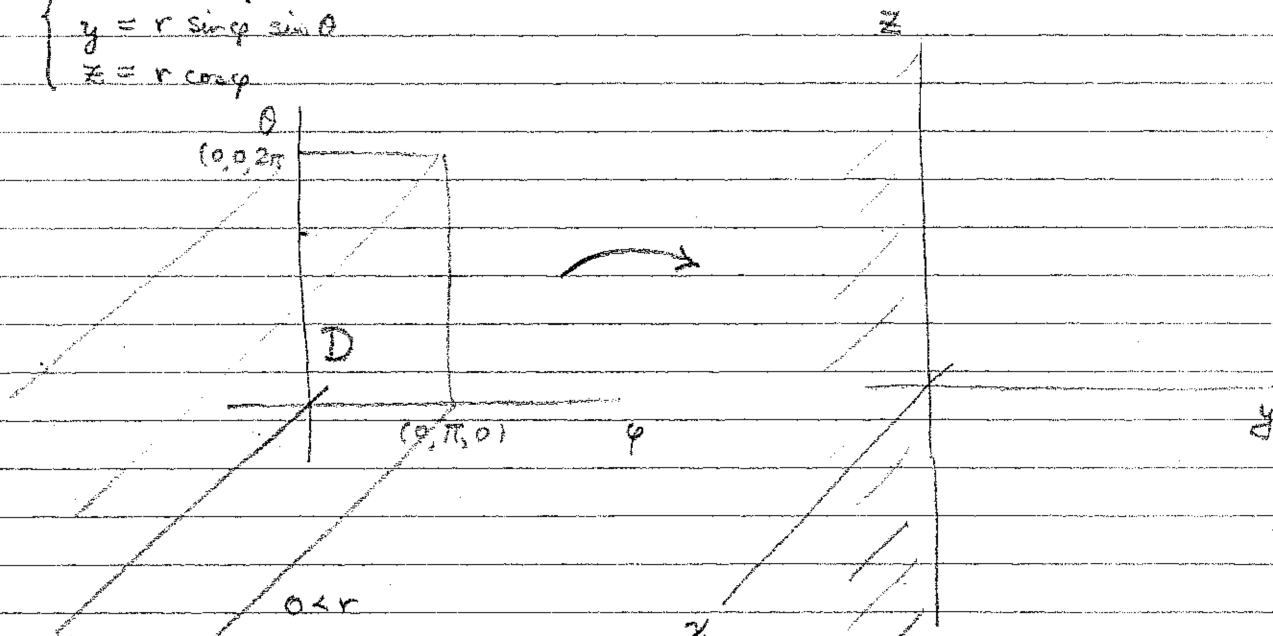
Math. 5C   Wednesday, May 11, 2011



$$\begin{cases} x = r\sin\varphi\cos\theta \\ y = r\sin\varphi\sin\theta \\ z = r\cos\varphi \end{cases}$$



$0 < r$

$0 < \varphi < \pi$

$0 < \theta < 2\pi$

front half of (x, z)-plane not covered.

$$\iiint_D e^{(x^2+y^2+z)^{\frac{3}{2}}}\,dx\,dy\,dz = ?$$

$$D = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 \leq 1\}$$

$$\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} & \frac{\partial z}{\partial \phi} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \end{vmatrix} = \cdots = \rho^2 \sin\phi \, d\rho \, d\varphi \, d\theta$$

$$\iiint_D e^{(x^2+y^2+z^2)^{3/2}}\,dx\,dy\,dz = \int_0^{2\pi}\left[\int_0^{\pi}\left[\int_0^1 e^{\rho^3}\rho^2 \sin\varphi \, d\rho\right] d\varphi\right] d\theta$$

$$= \int_0^{2\pi}\left[\int_0^{\pi} \frac{1}{3} e^{\rho^3}\Big|_0^1 \sin\varphi \, d\varphi\right] d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{3}(e-1)\sin\varphi \, d\varphi \, d\theta$$

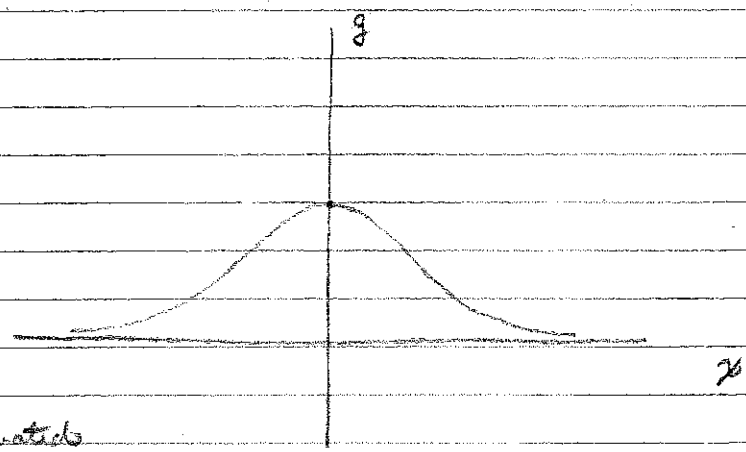
$$= \int_0^{2\pi} -\frac{1}{3}(e-1)\cos\varphi \Big|_0^{\pi} d\theta = \int_0^{2\pi} \frac{2}{3}(e-1)\,d\theta$$

$$= \boxed{\frac{4}{3}(e-1)}.$$

BELL-SHAPED CURVE

$$g(x) = e^{-\frac{x^2}{2\sigma^2}}$$

$$\int_0^{x_0} e^{-\frac{x^2}{2\sigma^2}}\,dx$$



cannot be evaluated

in terms of usual functions e^x $\sin x$ $\cos x$

$$I = \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}} dx$$

$$I^2 = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2\sigma^2}} dx \right] dy$$

$$= \int_0^{2\pi} \int_0^{\infty} e^{-\frac{r^2}{2\sigma^2}} r\,dr\,d\theta = \sigma^2 \int_0^{2\pi} \left[-e^{-\frac{r^2}{2\sigma^2}} \right]_0^{\infty} d\theta = 2\pi\sigma^2$$

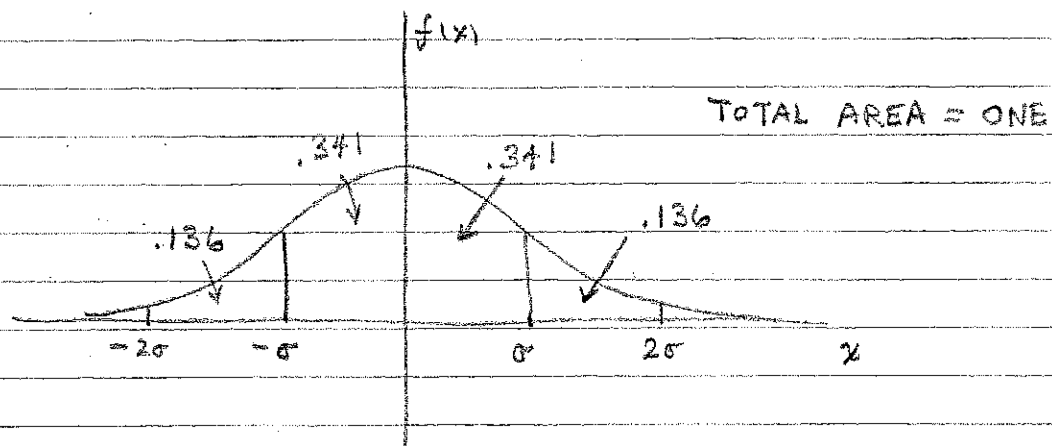
$$u = -\frac{r^2}{2\sigma^2} \qquad du = -\frac{r}{\sigma^2} dr$$

$$I = \sqrt{2\pi\sigma^2}$$

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma}} dx = \sqrt{2\pi\sigma^2}$$

Let $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}}$ $\qquad \int_{-\infty}^{\infty} f(x)\,dx = 1$

$f(x)$ is the <u>probability</u> <u>density</u> <u>function</u> for the NORMAL or GAUSSIAN distribution.



$$\mathrm{erf}\left(\frac{x}{\sigma}\right) = \frac{2}{\sqrt{2\pi}\,\sigma} \int_0^{x} e^{-\frac{t^2}{2\sigma^2}} dt$$