Math. 5B. Monday, March 28, 2011

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WebWork

http://homework.math.ucsb.edu/webwork2/

Username: Perm number

Password: Perm number

Change password after first login.

$\mathbb{R}^n = \{(x_1, \dots, x_n) : x_1, \dots, x_n \in \mathbb{R}\}$

$(2, 3, \pi, \sqrt{7}) \in \mathbb{R}^4$

Two vector space operations:

$(1, 3, 2, 6) + (2, 4, 0, 6) = \cdots$

$3(1, 2, 5, 0) = \cdots$

Dot product:

If $\vec{x} = (x_1, x_2, \dots, x_n)$ $\vec{y} = (y_1, y_2, \dots, y_n)$

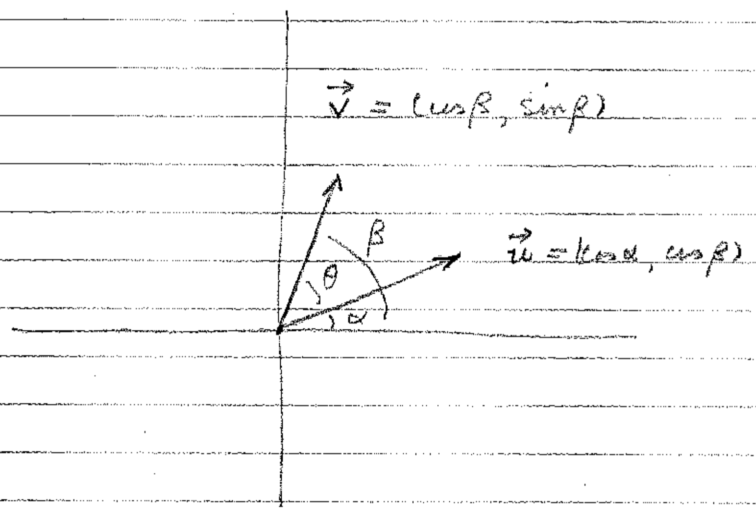
$\vec{x} \cdot \vec{y} = x_1 y_1 + \cdots + x_n y_n$.

$\|\vec{x}\| = \sqrt{\vec{x}\cdot\vec{x}}$

Dot product in $\mathbb{R}^2$:

Suppose $\vec{u} \in \mathbb{R}^2$, $\vec{v} \in \mathbb{R}^2$ $\|\vec{u}\| = \|\vec{v}\| = 1$.

$\vec{u} = (\cos\alpha, \sin\alpha)$, $\vec{v} = (\cos\beta, \sin\beta)$



$\theta = \beta - \alpha$

$\vec{u}\cdot\vec{v} = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\beta - \alpha) = \cos\theta.$

$\vec{u}\cdot\vec{v} = \cos\theta$ where $\theta = \angle$ between $\vec{u}$ & $\vec{v}$

$\vec{x}, \vec{y} \in \mathbb{R}^2$; $\vec{x}, \vec{y} \neq 0$ $\quad u = \dfrac{\vec{x}}{\|\vec{x}\|}$ $\quad \vec{v} = \dfrac{\vec{y}}{\|\vec{y}\|}$.

$\vec{u}\cdot\vec{v} = \cos\theta$, where $\theta = \angle$ between $\vec{x}$ & $\vec{y}$.

$\dfrac{\vec{x}}{\|\vec{x}\|} \cdot \dfrac{\vec{y}}{\|\vec{y}\|} = \cos\theta$ $\qquad \vec{x}\cdot\vec{y} = \|\vec{x}\|\,\|\vec{y}\|\cos\theta.$

In $\mathbb{R}^n$ we DEFINE the angle θ between $\vec{x}$ and $\vec{y}$ so that $\vec{x}\cdot\vec{y} = \|\vec{x}\|\,\|\vec{y}\|\cos\theta$.

What is $\angle$ between $\vec{x} = (2,1,1)$ and $\vec{y} = (1,1,0)$ in $\mathbb{R}^3$?

$\|\vec{x}\| = \sqrt{6}$ $\quad \|\vec{y}\| = \sqrt{2}$ $\quad \vec{x}\cdot\vec{y} = 3$

$$\cos\theta = \frac{\vec{x}\cdot\vec{y}}{\|\vec{x}\|\,\|\vec{y}\|} = \frac{3}{\sqrt{6}\sqrt{2}} = \frac{\sqrt{3}}{2} \qquad \theta = \frac{\pi}{6} = 30^\circ$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \cos\frac{\pi}{3} = \frac{1}{2}$$

SPECIAL CASE: $\vec{x} \perp \vec{y} \Leftrightarrow \vec{x}\cdot\vec{y} = 0$.

APPLICATION: Find equation of plane in $\mathbb{R}^3$ through $(0,0,0)$ which is $\perp$ to $(3,1,4)$

$(3,1,4)\cdot(x_1,x_2,x_3) = 0$

$3x_1 + x_2 + 4x_3 = 0$.

Standard basis for $\mathbb{R}^3$

$\vec{i} = (1,0,0)$, $\quad \vec{j} = (0,1,0)$ $\quad \vec{k} = (0,0,1)$

CROSS PRODUCT DEFINED ONLY IN $\mathbb{R}^3$

If $\vec{x} = (x_1, x_2, x_3)$ and $\vec{y} = (y_1, y_2, y_3)$

$$\vec{x} \times \vec{y} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

$$= \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} \vec{i} + \begin{vmatrix} x_3 & x_1 \\ y_3 & y_1 \end{vmatrix} \vec{j} + \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} \vec{j}.$$

IMPORTANT FEATURE OF CROSS PRODUCT

$\vec{x} \times \vec{y}$ is $\perp$ to $\vec{x}$ and $\vec{y}$.

Find equation of plane containing

$(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$.

$\vec{a} = (-1, 2, 0)$ and $b = (-1, 0, 3)$

are $\parallel$ to plane

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6\vec{i} + 3\vec{j} + 2\vec{k}$$

is $\perp$ to plane

$6(x-1) + 3y + 2z = 0.$