

Math 3CI: Project 1C
A model for infectious disease
Using EXCEL to find numerical solutions

October 9, 2009

You are the chief health officer on a small island which has a population of 50,000. An infectious disease has been spreading for some time and your task is to predict its future course. This disease is not fatal provided the patient's fever is kept down, and full recovery is usually expected. Once a person is infected, he may recover in a few days or several weeks, depending upon his ability to fight the disease off. About 5% of those sick will be expected to recover on any given day. Once recovered, the person is no longer infectious. According to your statistics, 2100 people are currently infected and 2500 have recovered.

A recent article in the New Guinea Journal of Medicine says that the rate of new infections for the disease perday can be calculated as $10^{-5}SI$, where S represents the number of people in the susceptible population and I represents the number of infected people. We can let R denote the number of recovered people in the population.

We could let t denote time in days, and let R be the number of people who have recovered. Then $S + I + R = 50,000$, so R can be determined once we know S and I . Our hypotheses lead to a system of differential equations

$$\begin{cases} \frac{dS}{dt} &= -10^{-5}SI, \\ \frac{dI}{dt} &= 10^{-5}SI - .05I. \end{cases} \quad (1)$$

Alternatively we could come up with a discrete time model

$$\begin{cases} S(t+1) &= S(t) - 10^{-5}S(t)I(t), \\ I(t+1) &= I(t) + 10^{-5}S(t)I(t) - .05I(t), \end{cases} \quad (2)$$

or if we wanted to consider changes over a smaller time interval h , we could use the discrete model,

$$\begin{cases} S(t+h) &= S(t) - h(10^{-5}S(t)I(t)), \\ I(t+h) &= I(t) + h(10^{-5}S(t)I(t) - .05I(t)), \end{cases} \quad (3)$$

where h is now a small number. Explain the relationship between models (1) and (3). (Hint: Even though the notion of limit may seem a little bit vague, think of the definition of derivatives in terms of limits.)

If you wanted to solve the system (1) directly, you could start by eliminating t between the two equations, and try to find the relation between S and I . How far can you get with this approach? Can you graph I as a function of S ? Can you find an integral expression for $I(t)$? For the given initial conditions $I(0) = 2100$ and $R(0) = 2500$, can you determine whether everybody will become infected? What is the maximum number of people infected at a given point in time?

If you use model two, you could determine $S(t)$ and $I(t)$ for $t = 1, 2, \dots, 10$. What do you obtain for $S(10)$ and $I(10)$? You might want to carry the calculations out with a calculator or with an Excel spreadsheet.

Formula (3) is known as the *Cauchy-Euler polygon* method for finding approximate solutions to (1). It is the simplest of several numerical methods for solving differential equations. Such numerical methods are discussed in upper division mathematics courses such as Math 104.

Here is an explanation of how to set up an excel spread sheet (on a Mac) to do the numerics:

1. In Row 1 of the spreadsheet you provide labels for the columns, the stage n , the time t_n and the values $S(t_n)$ and $I(t_n)$. (You could also include a column for $R(t_n)$ if you want.)

2. In Row 2, you enter the initial conditions.

3. In Row 3, Column A you enter $A2 + 1$ then hit the enter key. You fill in the next three entries of Row 3 with

$$= B2 + 1 \quad = C2 - .00001 * C2 * D2 \quad = D2 + .00001 * C2 * D2 - .05 * D2$$

each time hitting the enter key.

4. You then select Rows 4 through 42 and from the “Edit” menu at the top of the screen, select ”Fill Down.”

This will give you a spreadsheet that will provide 40 iterations of model (2). You could modify it to consider other choices of initial conditions or choose $h = .1$ and apply model (3), which would give you a better approximation to the solutions to the system of differential equations (1). You can also modify the spreadsheet to provide numerical solutions to the Volterra-Lotka predator-prey equations, or other systems of equations treated later in the course.