MAT 116 In-class Problems (#4)

June 30, 2010 and July 1, 2010

Pidgeonhole Principle (Simple Version). *If* $n+1$ *objects are distributed into* n *boxes, then at least one box contains two or more of the objects.*

Application 1. Among 13 people there are 2 who have their birthdays in the same month.

Application 2. There are n married couples. How many of the $2n$ people must be selected to guarantee that a married couple has been selected?

Application 3. Given m integers $a_1, a_2, \ldots, a_m$, there exist integers k and l with $0 \leq k < l \leq m$ such that $a_{k+1} + \cdots + a_l$ is divisible by m. Less formally, there exist consecutive a's in the sequence $a_1, a_2, \ldots, a_m$ whose sum is divisible by m.

Application 4. A chess master who has 11 weeks to prepare for a tournament decides to play at least one game every day but, to avoid tiring himself, he decides not to play more than 12 games during any calendar week. Show that there exists a succession of (consecutive) days during which the chessmaster will have played *exactly* 21 games.

Application 5. From the integers $1, 2, \ldots, 200$, we choose 101 integers. Show that, among the integers chosen, there are two such that one of them is divisible by the other.

Application 6. (*Chinese remainder theorem*) Let m and n be relatively prime positive integers, and let a and b be integers where $0 \leq a \leq m-1$ and $0 \leq b \leq n-1$. Then there is a positive integer x such that the reminder when x is divided by m is a, and the remainder when x is divided by n is b; that is, x can be written in the form $x = pm + a$ and also in the form $x = qn + b$ for some integers p and q.