

MAT 116 In-class Problems (#2)

June 22, 2010 and June 23, 2010

- How many distinct positive divisors does each of the following numbers have?
 - $3^4 \cdot 5^2 \cdot 7^6 \cdot 11$
 - 620
 - 10^{10}
- What is the number of ways to order the 26 letters of the alphabet so that no two of the vowels a, e, i, o, u occur consecutively?
- Ten people, including two who do not want to sit next to one another, are to be seated at a round table. How many circular seating arrangements are there?
- Prove **Pascal's formula**:

$$\text{For all integers } n \text{ and } k \text{ with } 1 \leq k \leq n - 1, \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

in each of the following ways:

- By using the formula $\binom{m}{r} = \frac{m!}{r!(m-r)!}$ and doing some algebra.
- By using a combinatorial argument. Interpret $\binom{m}{r}$ as the number of r -element subsets of an m -element set. As a warm up, do this for the case $n = 4$ and $k = 2$.