On deriving priority vectors from multiple pairwise comparison matrices

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The problem of importance weight analysis and determination from multiple source information is a critical issue in many fields such as decision making, machine learning, meta-search engines, etc. We focus on the problem of computing the importance weights and the corresponding rank ordering of a set of alternatives from information given by a group of experts into the form of pairwise comparison matrices, in a context of group decision theory.

Let $X = \{x_1, ..., x_n\} (n \geq 2)$ be a finite set of alternatives. A pairwise comparison matrix on the set X is an $n \times n$ positive matrix $A = (a_{ij})$, where a_{ij} denotes the ratio of the preference intensity of alternative x_i to that of x_j. A is assumed to be reciprocal, that is $a_{ij}a_{ji} = 1$ for $i, j = 1, \ldots, n$. A pairwise comparison matrix A is said to be consistent if $a_{ij}a_{jk} = a_{ik}$ for $i, j, k = 1, \ldots, n$. If the pairwise matrix A is consistent, then there exists a positive vector $w = (w_1, \ldots, w_n)^T$ such that

$$a_{ij} = \frac{w_i}{w_j} \qquad i, j = 1, \ldots, n. \tag{1}$$

Moreover the vector w is unique up to a multiplicative constant and it is an eigenvector associated to the largest eigenvalue (n) of the matrix A. The elements of w, normalized, define the searched weights of the alternatives. Otherwise, if matrix A is not consistent, which is the most usual in applications, it does not exists exact values w_i verifying (1). In this case, the problem is to determine the positive weight vector w that best reflect preference information in matrix A. This problem has attracted considerable research interest and several priorization methods have been proposed in the literature. An interesting approach (distance-based methods) consists on establish the problem in a framework of matrix approximation where an optimization problem is established.

In practice, decision-making problems are faced the lack of information, imprecise data and conflicting multiple expert preferences. These features

pose new challenges and call for new techniques dealing with multiple and imprecise information expressed into a set of interval or fuzzy pairwise comparison matrices.