

BRAIDS AND DIAGRAMMATIC ALGEBRAS.

We will investigate various groups, monoids, and other algebraic structures, whose elements are represented by certain kinds of two-dimensional diagrams drawn inside a rectangle. This will continue the research from the 2011 REU which has resulted in a preprint “The Alexander and Jones Polynomials through representations of rook algebras” and “A presentation of the Motzkin algebra” (work in preparation).

A braid is a certain kind of arrangement of pieces of string in space. The endpoints of the strands are lined up at the top and the bottom of the braid. The set of braids with a given number of strands forms a group, where “multiplication” is defined by stacking one braid on top of the other and joining up the strands.

A braid is a three-dimensional object, but we can represent it by a two dimensional picture that includes “crossings” where one strand goes over or under another. Recently there has been increasing interest in other algebras defined using two dimensional pictures. These have applications including von Neumann algebras, and Feynman diagrams in physics.

Our goal is to define and investigate new algebras of this kind. The diagrams should not include crossings, but can introduce features such as coloring or orientations on the strands. We are especially interested in maps from the braid group to these algebras. Such maps can simplify a braid by eliminating its crossings, and ultimately give invariants of knots and other topological objects made of strings.