

THE PROBLEM SOLVING COMPETITION
NOVEMBER 15, 2007

Problem

Let

$$b(x) = (x - a)^2, \quad 0 \leq x \leq 1, \quad 0 \leq a \leq 1.$$

Denote the mean value of the function $b(x)$ on the interval from s to t by the formula

$$M(s, t) = \frac{1}{t - s} \int_s^t b(x) dx, \quad 0 \leq s \leq 1, \quad 0 \leq t \leq 1.$$

Show that

$$M(s, t) \geq \frac{b(t)}{4}, \quad \text{for all } 0 \leq s \leq 1, \quad 0 \leq t \leq 1.$$