**Homework #2: Solutions**

1(a) Suppose $|b_n| \leq k$ for all $n$. Given $\epsilon > 0$, there is an $N$ so that $|a_n| < \epsilon k$ for all $n \geq N$. Then for $n \geq N$,

$$|a_n b_n| \leq k \cdot |a_n| < k \cdot \epsilon k = \epsilon.$$

(b) FALSE. Take $a_n = 1$ for all $n$ & $b_n = (-1)^n$.

2(a) If $A > 0$, then take $\epsilon = |A|/10$. There is an $N$ so that $|a_n - A| < \epsilon$ for $n \geq N$, but then $a_n < A$ for those $n$. So $A \geq 0$.

(b) Take $a_n = 1/n$. Then $a_n > 0$ but $A = 0$.

(c) Since $a_n \leq |a_n|$, $b_n = |a_n| \geq 0$ & $b_n \to |a_n|$. By (a), $|a_n - A| < \epsilon$ for $n \geq N$, similarly other part.

3(a) Fix $p$. Then given $\epsilon > 0$, there is an $N$ so that $|a_n - A| < \epsilon$ for all $n \geq N$. In particular this works for all $n \geq N$, so $|a_{np} - A| < \epsilon$ for $np \geq N$.

(b) FALSE. Take $a_n = 0$ \( n \) not prime

Then for every $p > 1$ the sequence \( \{a_{np}\} \) contains at most one prime number (in the first), so $a_{np} \to 0$.

But $\{a_n\}$ doesn't converge; there are infinitely many primes.