1. Given $\varepsilon > 0$, since $b_n \to 0$, there is a $N$ so that $|b_n| < \varepsilon$. Moreover, since $|l_{n+1}-l_n| \leq b_n$ for large $n$, say for $n \geq M$, we have $0 \leq b_n$ for $n \geq M$. Putting these together, for $n \geq \max(N, M)$

$|l_{n+1}-l_n| \leq b_n < \varepsilon.$

2. (i) See Math 8

(ii) Suppose $\{x_n\}$ converges, the only possible limits are $\{0, 1, \ldots, 9\}$ since these are the only values. Suppose $\alpha_n \to A$ say. Take $\varepsilon = \frac{1}{10}$, then there is an $N$ so that $|\alpha_n - A| < \frac{1}{10}$ for $n \geq N$ & we deduce $\alpha_n = A$ for $n \geq N$.

In other words $\sqrt{2} = 1.414\ldots \neq A A A \ldots$

So $\sqrt{2} = \frac{\text{some other number}}{10^3} + \frac{1}{10^3} \times 0.\overline{AAA}$

is rational. $\Box$

Remark (i) is the important part. It needs clear argument.

3. FALSE. Take $\alpha_n = \log(n)$.

Then for every fixed $p$

$|\alpha_{n+p} - \alpha_n| = |\log(n+p) - \log(n)| = |\log\left(\frac{n+p}{n}\right)|$

$= |\log\left(1 + \frac{p}{n}\right)| \to 0$ as $n \to \infty$.

But $\alpha_n$ does not converge. $\Box$