Math 108A Homework No. 8

1. (a) Suppose that $T \in \mathcal{L}(V)$ is an operator with $T^n = 0$. Prove that I - T is invertible.

(b) Give an example of an operator with $T^3 = 0$ but $T^2 \neq 0$.

(c) True or false: I + T must also be invertible.

2. Suppose that $T \in \mathcal{L}(V)$ has $T^2 = T$. Prove that $V = ker(T) \oplus Im(T)$. Give an example of such an operator which is not the identity operator.

3. V is finite dimensional and $T \in \mathcal{L}(V)$. Prove the following are equivalent: (a) $ker(T) \oplus Im(T) = V$ (b) ker(T) + Im(T) = V(c) $ker(T) \cap Im(T) = \{0\}$