## Math 108A Homework No. 8

1. (a) Suppose that $T \in \mathcal{L}(V)$ is an operator with $T^{n}=0$. Prove that $I-T$ is invertible.
(b) Give an example of an operator with $T^{3}=0$ but $T^{2} \neq 0$.
(c) True or false: $I+T$ must also be invertible.
2. Suppose that $T \in \mathcal{L}(V)$ has $T^{2}=T$. Prove that $V=\operatorname{ker}(T) \oplus \operatorname{Im}(T)$. Give an example of such an operator which is not the identity operator.
3. $V$ is finite dimensional and $T \in \mathcal{L}(V)$. Prove the following are equivalent:
(a) $\operatorname{ker}(T) \oplus \operatorname{Im}(T)=V$
(b) $\operatorname{ker}(T)+\operatorname{Im}(T)=V$
(c) $\operatorname{ker}(T) \cap \operatorname{Im}(T)=\{0\}$
