Math 108A Homework No. 7

1. Suppose that $T \in \mathcal{L}(V)$ has that the $dim(Im(T)) = k$. Prove that T has at most $k+1$ eigenvalues.

2. Suppose that $V = A \oplus B$ and define an operator on V by the rule $P(a+b) = a$. Find all eigenvalues and eigenvectors of P.

3. Suppose S and T are operators on V and that S is invertible.
(a)Prove that T and $S^{-1}.T.S$ have the same eigenvalues.
(b) Describe the connexion between the eigenvectors of T and those of $S^{-1}.T.S$.

4. Suppose that S and T are operators on V. Show that ST and TS have the same set of eigenvalues. (Warning: Be careful not to assume that either S or T is invertible.)