## Math 108A Homework No. 6

1. Suppose that $\phi \in \mathcal{L}(V, \mathbf{R})$ and $\mathbf{u}$ is not in $\operatorname{ker}(\phi)$. Prove that

$$
V=\operatorname{ker}(\phi) \oplus\{\lambda \mathbf{u} \mid \lambda \in \mathbf{R}\}
$$

2. Suppose that $T \in \mathcal{L}(U, V)$ and $S \in \mathcal{L}(V, W)$ are both invertible. Prove that $S T \in \mathcal{L}(U, W)$ is invertible and $(S T)^{-1}=T^{-1} S^{-1}$.
3. Suppose that $V$ and $W$ are finite dimensional and let $\mathbf{v} \in V$. Let

$$
E=\{T \in \mathcal{L}(V, W) ; \mid T(\mathbf{v})=\mathbf{0}\}
$$

Show that $E$ is a subspace of $\mathcal{L}(V, W)$. If $\mathbf{v} \neq \mathbf{0}$, what is the dimension of $E$ ?
4. Suppose that $V$ is finite dimensional and that $S$ and $T$ are in $\mathcal{L}(V)$. Prove that $S T=I$ if and only if $T S=I$.

