Math 108A Homework No. 6

1. Suppose that $\phi \in \mathcal{L}(V, \mathbf{R})$ and **u** is not in $ker(\phi)$. Prove that

$$V = ker(\phi) \oplus \{\lambda \mathbf{u} \mid \lambda \in \mathbf{R}\}\$$

2. Suppose that $T \in \mathcal{L}(U, V)$ and $S \in \mathcal{L}(V, W)$ are both invertible. Prove that $ST \in \mathcal{L}(U, W)$ is invertible and $(ST)^{-1} = T^{-1}S^{-1}$.

3. Suppose that V and W are finite dimensional and let $\mathbf{v} \in V$. Let

$$E = \{T \in \mathcal{L}(V, W); \mid T(\mathbf{v}) = \mathbf{0} \}$$

Show that E is a subspace of $\mathcal{L}(V, W)$. If $\mathbf{v} \neq \mathbf{0}$, what is the dimension of E?

4. Suppose that V is finite dimensional and that S and T are in $\mathcal{L}(V)$. Prove that ST = I if and only if TS = I.