Math 108A Homework No. 5

1. Suppose that V is a vector space and $S, T \in \mathcal{L}(V, V)$ satisfy $range(S) \subset null(T)$. Prove that $STST$ is the zero map.

2. (a) Give an example of a linear map $T : \mathbf{R}^4 \longrightarrow \mathbf{R}^4$ with $range(T) = null(T)$.
(b) Prove or disprove: There is no such map $T : \mathbf{R}^5 \longrightarrow \mathbf{R}^5$.

3. (a) Suppose V and W are both finite-dimensional. Prove that there exists an injective linear map from V to W if and only if $dimV \leq dimW$.
(b) Suppose V and W are both finite-dimensional. Prove that there exists an surjective linear map from V to W if and only if $dimV \geq dimW$.

4. Suppose that U and V are finite dimensional vector spaces and $T \in \mathcal{L}(U, V)$ and $S \in \mathcal{L}(V, W)$. Prove

$$dim(null(ST)) \leq dim(null(S)) + dim(null(T))$$