Math 108A Homework No. 4

1. Prove or give a counter-example: If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a basis of V and $U \leq V$ such that $\mathbf{v}_1, \mathbf{v}_2$ are both in U and $\mathbf{v}_3$ and $\mathbf{v}_4$ and both not in U, then $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis of U.

2. Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_m\}$ are linearly independent and $\mathbf{w} \in V$. Prove that

$$dim(span(\mathbf{v}_1 + \mathbf{w}, \mathbf{v}_2 + \mathbf{w}, ...\mathbf{v}_m + \mathbf{w})) \geq m - 1$$

3. Suppose that $U_1, ..., U_k$ are subspaces of V. Prove that

$$dim(U_1 + U_2 + ... + U_k) \leq dim(U_1) + dim(U_2) + ... + dim(U_k)$$

4. Find all values of λ so that the vectors $(\lambda, 1, 1), (1, \lambda, 1), (1, 1, \lambda)$ are linearly dependent.