Math 108A Homework No. 3

1. Prove carefully that any non-empty subset of a linearly independent set of vectors is itself linearly independent.

2. (i) Prove carefully that if subsets S_1 and S_2 satisfy $S_1 \subset S_2$, then

$$span(S_1) \subset span(S_2)$$

(ii) Prove carefully that if S_1 and S_2 are subsets of V then

$$span(S_1 \cup S_2) = span(S_1) + span(S_2)$$

3. Show that the union of two subspaces is a subspace if and only if one contains the other.

4. Suppose that $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k$ are linearly independent and $\mathbf{w} \in V$.
Prove that $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k, \mathbf{w}$ are linearly independent if and only if $\mathbf{w} \notin span(\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k)$