Math 108A Homework No. 2

1.Prove carefully that if $U, V \leq W$, then $U + V \leq W$.
Is this still true if U is only a nonempty subset of W?

2. (a) Give an example of a subset U of $\mathbf{R}^2$ which is closed under addition and has additive inverses, but is not a subspace of $\mathbf{R}^2$.
(b) Give an example of a subset U of $\mathbf{R}^2$ which is closed under scalar multiplication but is not a subspace of $\mathbf{R}^2$.

3. Prove or disprove:
(a) $U \leq V$ then $U + U = U$.
(b) A, B, C all subspaces of V, then $A \oplus B = A \oplus C$ implies $B = C$.

4. Prove or disprove:
(a) If the vectors $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k$ are linearly independent and λ_1, ... λ_k is a list of nonzero scalars, then the vectors $\lambda_1\mathbf{v}_1, \lambda_2\mathbf{v}_2, ..., \lambda_k\mathbf{v}_k$ are linearly independent.
(b) If $\{\mathbf{v}_1, \mathbf{v}_2, ..., .\mathbf{v}_k\}$ and $\{\mathbf{w}_1, \mathbf{w}_2, ..., .\mathbf{w}_k\}$ are both linearly independent lists of vectors, then $\{\mathbf{v}_1 + \mathbf{w}_1, \mathbf{v}_2 + \mathbf{w}_2, ..., .\mathbf{v}_k + \mathbf{w}_k\}$ is a linearly independent list.