Math 108A Homework No. 1

- **1.** Prove that $-(-\mathbf{v}) = \mathbf{v}$.
- **2.** Suppose that $\lambda \in \mathbf{R}$ and $\mathbf{v} \in V$ satisfy $\lambda \mathbf{v} = \mathbf{0}$. Prove that either $\lambda = 0$ or $\mathbf{v} = \mathbf{0}$.
- **3.** Prove carefully that the subset of \mathbf{R}^{∞}

 $\{(x_1, x_2, x_3, \dots) \mid x_i \text{ is a nonzero real number for only finitely many } i\}$

is a real vector space (with the obvious operations).

4. Prove carefully that a non-empty subset U of a vector space V is a subspace if and only if for every $\lambda, \mu \in \mathbf{R}$ and \mathbf{v} and \mathbf{w} in U, the linear combination $\lambda \mathbf{v} + \mu \mathbf{w} \in U$.