Math 164: Homework 7.5

(Not to be turned in: extra practice for midterm)

Questions followed by * are to be turned in. Questions without * are extra practice. At least one extra practice question will appear on each exam.

Question 1 (Textbook Problem 3.2.1 (i))

Compute a basis matrix for the null space of the matrix A and express the x as $x = p + q$ where p is in the null space of A and q is in the range of A^T.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} , \quad x = (-2, 4, 5, -2)^T .$$

Question 2 (Similar to Textbook Problem 3.2.1 (ii))

Compute a basis matrix for the null space of the matrix A and express the x as $x = p + q$ where p is in the null space of A and q is in the range of A^T.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix} , \quad x = (3, 4, 0, 4)^T .$$

Question 3 (Textbook Problem 11.2.1(i))

Use the first and second derivatives to find the local maxima and local minima of

$$f(x) = 15 + 12x - 25x^2 + 2x^3 .$$

Question 4 (Similar to Textbook Problem 11.2.3)

Consider the function

$$f(x_1, x_2) = 8x_1^2 + 4x_1x_2 + 12x_2^2 - 24x_1 + 40x_2 - 28 .$$

Find all stationary points of this function and determine whether they are local minimizers and maximizers. Does this function have a global minimizer or a global maximizer? (Hint: when do you know that a local minimizer is a global minimizer?)

Question 5 (Textbook Problem 11.2.9)

Let

$$f(x) = 2x_1^2 + x_2^2 - 2x_1x_2 + 2x_1^3 + x_1^4 .$$

Determine the minimizers/maximizers of f and indicate what kind of minima or maxima (local, global, strict) they are.

Question 6 (Similar to Textbook Problem 11.2.10)

Let

$$f(x) = cx_1^2 + 2x_2^2 - 4x_1x_2 - 4x_2 + 2 ,$$

where $c \in \mathbb{R}$.

(a) Determine the stationary points of f for each value of c.

(b) For what values of c can f have a minimizer?