

- 7.1 (a) Let $p=13$. Compute $L_2(3)$**
(b) Show that $L_2(11) = 7$

(a) & (b) We want to find x such that $2^x \equiv 7 \pmod{13}$. Note first that 2 is a primitive root mod 13 because $2^6 = 64 \equiv -1 \pmod{13}$. Taking successive powers of 2, we get $2^4 = 16 \equiv 3 \pmod{13}$. So we have that $x=4$, and $L_2(3) = 4$. Next, we can check that $2^7 = 128 \equiv 11 \pmod{13}$. No other power between 1 and 12 works because 2 is a primitive root (mod 13). Thus, we have that $L_2(11) = 7$.

For larger moduli we would want to reduce the number of cases we have to compute using, for example, the technique in section 7.2 to first determine whether x is even or odd.