

Chapter 6 number 23:

If $\gcd(e, 12345)=1$ then we can find a number d such that $ed \equiv 1 \pmod{12345}$; that is, there exists an integer k with $ed = k \cdot 12345 + 1$. Then we will raise m^e to the d power $(m^e)^d = m^{k \cdot 12345 + 1} = m \cdot m^{k \cdot 12345} \equiv m \pmod{n}$ since $m^{k \cdot 12345} = (m^{12345})^k \equiv 1 \pmod{n}$. Thus, I would be able to find m in this case since the spy gave the information $m^{12345} \equiv 1 \pmod{n}$. In practice $\gcd(e, 12345) = 1$ will occur quite often as e is often chosen to be a large prime such as $2^{16}+1$, to ensure that it will be relatively prime with $\phi(n)$. $\gcd(e, 12345)=1$ if 12345 is the order of $m \pmod{n}$ (this is not necessarily the case in general), as in this case you will have 12345 divides $\phi(n)$ and we also know that $\gcd(\phi(n), e)$ will be 1.

Note that the prime factorization of 12345 is $3 \cdot 5 \cdot 823$