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Math 2D: Quiz 2

(5) 1. Find the traces of the following surface, then identify the surface. $16x^2 = y^2 + 4z^2$

Let k be a constant, if $x = k$ then we have the equation

$$y^2 + 4z^2 = 16k^2 \quad \Rightarrow \quad \frac{y^2}{(4k)^2} + \frac{z^2}{(2k)^2} = 1$$

So the traces on the zy-plane are a family of ellipses. If $y = k$, then we have

$$16x^2 - 4z^2 = k^2 \quad \Rightarrow \quad \frac{x^2}{(k/4)^2} - \frac{z^2}{(k/2)^2} = 1, \quad k \neq 0$$

So the traces on the xz-plane are a family of hyperbolas if $k \neq 0$, and two intersecting lines if $k = 0$. If $z = k$, then we have

$$16x^2 - y^2 = 4k^2 \quad \Rightarrow \quad \frac{x^2}{(k/2)^2} - \frac{y^2}{(2k)^2} = 1, \quad k \neq 0$$

So the traces on the xy-plane are a family of hyperbolas if $k \neq 0$, and two intersecting lines if $k = 0$. Putting te traces together we have an elliptic cone opening on the x-axis and vertex at the origin.

(5) 2. Find the equation for the surface consisting of all points that are equidistant from the point $(-1, 0, 0)$ and the plane $x = 1$, then identify the surface.

Let $P = (x, y, z)$ be an arbitrary point equidistant from $(-1, 0, 0)$ and the plane $x = 1$. First take note that the normal for the plane is $\langle 1, 0, 0 \rangle$. Now then the distane D from P to $(-1, 0, 0)$ is given by

$$D = \sqrt{(x+1)^2 + y^2 + z^2}$$

and the distance from the point P to the plane $x = 1$ is

$$D = \frac{|x - 1|}{\sqrt{1^2 + 0^2 + 0^2}} = |x - 1|$$

equating the two we have

$$|x - 1| = \sqrt{(x+1)^2 + y^2 + z^2} \quad \Rightarrow \quad (x-1)^2 = (x+1)^2 + y^2 + z^2 \quad \Rightarrow \quad -4x = y^2 + z^2$$

The equation of such points is $-4x = y^2 + z^2$, which is a circular paraboloid with vertex at the origin, and opening on the negative x-axis.