

Name: _____

Signature: _____
Winter 2009 Math 2D Final Exam

1. Let

$$f(x, y) = e^{x^2 - y^2}$$

- (a) (5pts) What is the directional derivative of f in the direction of $\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$ at the point $(2, 3)$?
- (b) (5pts) What is the direction of greatest increase of f at the same point $(2, 3)$?
(Give the direction as a unit vector.)

2. (5 pts) Let

$$f(x, y) = x^2 + y^2 - x - y + 5$$

Find the absolute extreme of f on the region bounded by the lines $x = 0, y = 0, x + y = 3$

3. (5 pts) Suppose $z = f(x, y)$ is a (differentiable) function of two variables and $\mathbf{r}(t) = (x(t), y(t))$ is a (differentiable) plane curve. Assume

$$\nabla f(1, 2) = \langle 4, 5 \rangle \quad \mathbf{r}(3) = (1, 2) \quad \mathbf{r}'(1, 2) = \langle 2, 3 \rangle.$$

Define $g(t) = f(\mathbf{r}(t))$. What is $g'(3)$?

4. (5 pts) Use Lagrange multipliers to write down three equations in three unknowns that would let you find the maximum value of

$$H(x, y) = e^{x^2 - 3y^3}$$

on the unit circle $2x^2 + 3y^2 = 1$. (Do not bother solving the equations or finding the maximum value.)

5. Let R be the region bounded by the graphs of $y = x^2$ and $x + y = 1$. Write the integral

$$\iint_R f(x, y) dA$$

as iterate integral

- (a) (5 pts) First in y and after in x .

- (b) (5 pts) First in x and after in y .

6. (5 pts) Computes the surface area of the graph of

$$f(x, y) = y^2$$

over the region in the xy -plane enclosed by the lines

$$y = 0 \quad y = 3 \quad x = y.$$

7. (5 pts) Setup $\iiint_Q f(x, y, z) dV$ over the solid region Q bounded by the graphs of

$$z = 6 - y^2, x = y^2, y = 0, z = 2.$$

8. (5 pts) Setup $\iiint_Q f(x, y, z) dV$ using cylindrical coordinates where Q is the solid region

$$x^2 + y^2 \leq 3, 0 \leq z \leq 3 + x^2, 0 \leq y \leq x.$$