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Math 2D: Quiz 1 solutions

(5) 1. Find the unit direction for the line of intersection of the planes $x + y + z = 1$ and $x + z = 0$.

Setting $x = 0$, we see that $(0, 1, 0)$ satisfies the equations of both planes, so that they do in fact have a line of intersection. The respective normal vectors are $n_1 = \langle 1, 1, 1\rangle$, $n_2 = \langle 1, 0, 1\rangle$. So

$$v = n_1 \times n_2 = \langle 1, 1, 1\rangle \times \langle 1, 0, 1\rangle = \langle 1, 0, -1\rangle$$

Therefore the direction of the line is $\langle 1, 0, -1\rangle$, and the unit direction is $\frac{1}{\sqrt{2}}\langle 1, 0, -1\rangle$.

(5) 2. Find the parametric equations for the line through the point $(0, 1, 2)$ that is perpendicular, and intersects the line $L = \{x = 1 + t, y = 1 - t, z = 2t\}$.

First writing L in vector form we have $\langle x, y, z\rangle = \langle 1, 1, 0\rangle + t\langle 1, -1, 2\rangle$. So $v = \langle 1, -1, 2\rangle$. Now if $t = 0$ then a point on L is $(1, 1, 0)$. So we have another vector $w = (0, 1, 2) - (1, 1, 0) = \langle -1, 0, 2\rangle$. Then

$$w - proj_v(w) = w - \frac{v \cdot w}{\|v\|^2}v = \langle -1, 0, 2\rangle - \frac{\langle 1, -1, 2\rangle \cdot \langle -1, 0, 2\rangle}{6}\langle 1, -1, 2\rangle = \langle -1, 0, 2\rangle - \frac{1}{2}\langle 1, -1, 2\rangle = \frac{1}{2}\langle -3, 1, 2\rangle$$

So a direction vector for the line we want is $\langle -3, 1, 2\rangle$. So our vector equation is

$$\langle x, y, z\rangle = \langle 0, 1, 2\rangle + t\langle -3, 1, 2\rangle$$

hence the parametric equations are

$$x = -3t, y = 1 + t, z = 2 + 2t$$