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Math 2D: Quiz 5

(5) **1.** Find the extreme values of $f(x,y) = e^{-xy}$, on the region $\mathcal{R} = \{x^2 + 4y^2 \leq 1\}$

For the interior of the region we find the critical points:

$$f_x = -ye^{-xy}, \quad f_y = -xe^{-xy}$$

then $f_x = f_y = 0$ implies that $x = y = 0$, so $(0,0)$ is the only critical point and $f(0,0) = 1$. For the boundary, let $g(x,y) = x^2 + 4y^2 = 1$, then by Lagrange multipliers we have

$$\langle -ye^{-xy}, -xe^{-xy} \rangle = \lambda \langle 2x, 8y \rangle$$

so our system is

$$\begin{aligned} -ye^{-xy} &= \lambda 2x \\ -xe^{-xy} &= \lambda 8y \\ x^2 + 4y^2 &= 1 \end{aligned}$$

solving for $-e^{-xy}$ in the first equation and plugging into the second we have

$$-e^{-xy} = \frac{\lambda 2x}{y} \quad \Rightarrow \quad x\frac{\lambda 2x}{y} = \lambda 8y \quad \Rightarrow \quad x^2 = 4y^2$$

Plugging this relationship into the constraint we have

$$x = \pm\frac{1}{\sqrt{2}}, \quad y = \pm\frac{1}{2\sqrt{2}}$$

Plugging these 4 points into our function we find that

$$f\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{2\sqrt{2}}\right) = e^{-1/4}, \quad f\left(\pm\frac{1}{\sqrt{2}}, \mp\frac{1}{2\sqrt{2}}\right) = e^{1/4}$$

so $\left(\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$, $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right)$ are the minimum with a value of $e^{-1/4}$, and $\left(\frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right)$, $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$ are the maximum with a value of $e^{-1/4}$.

(5) **2.** Evaluate $\iint_{\mathcal{R}} e^{x/y}\; dA$ over the give region $\mathcal{R} = \{(x,y) : y \leq x \leq y^3, 1 \leq y \leq 2\}$

Over the region our integral is

$$\begin{aligned} \int_1^2 \int_y^{y^3} e^{x/y}\; dxdy &= \int_1^2 ye^{x/y}\Big|_{x=y}^{x=y^3} dy \\ &= \int_1^2 ye^{y^2} - ey\; dy \\ &= \frac{e^{y^2}}{2} - \frac{ey^2}{2}\Big|_1^2 = \frac{1}{2}(e^4 - 4e) \end{aligned}$$