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Math 2D: Quiz 4

1. A box without a lid is to have a volume of $32cm^3$. Find the dimensions that minimize the amount of cardboard used.

Let (x, y, z) be the dimensions of the box. Then we know that the volume is $xyz = 32$ and we want to minimize the surface area, which is given by the equation $f(x, y, z) = xy + 2z(x + y)$. From the volume let $z = \frac{32}{xy}$ then define a function $g(x, y)$ as follows.

$$g(x, y) = f(x, y, \frac{32}{xy}) = xy + \frac{64}{xy}(x + y)$$

Computing the partials we have

$$g_x = y - \frac{64}{x^2}, \quad g_y = x - \frac{64}{y^2}$$

So $g_x = 0$ implies that $y = \frac{64}{x^2}$, plugging this into $g_y = 0$ we have

$$x - \frac{64}{(\frac{64}{x^2})^2} = 0 \quad \Rightarrow \quad x(x^3 - 64) = 0$$

But $x \neq 0$. So $x^3 - 64 = 0$, hence $x = 4$. Then we have

$$y = \frac{64}{4^2} = 4, \quad z = \frac{32}{4(4)} = 2$$

Now we need to check if the value is actually a minimum. So

$$f_{xx} = 2\frac{64}{x^3}, \quad f_{yy} = 2\frac{64}{y^3}, \quad f_{xy} = 1$$

So $D(x, y) = 4\frac{64^2}{x^3y^3} - 1$ and $D(4, 4) = 4 - 1 = 3 > 0$ and $f_{xx}(4, 4) = 2 > 0$. So (4,4,2) is indeed a minimum, and so the dimension are $(4, 4, 2)cm$.

2. Find the maximum rate of change of $f(x, y) = \dfrac{y^2}{x}$ at $p = (2, 4)$ and the direction in which it occurs.

$$\nabla f = \langle -\frac{y^2}{x^2}, \frac{2y}{x} \rangle \quad \Rightarrow \quad \nabla f(2, 4) = \langle -4, -4 \rangle$$

so the direction of the maximum rate of change at $(2, 4)$ is $\langle -4, -4 \rangle$ and the maximum rate of change is given by

$$\|\nabla f(2, 4)\| = \|\langle -4, -4 \rangle\| = 4\sqrt{2}$$