Name:
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Math 2D: Quiz 6

(5) **1.** Evaluate $\iint_{\mathcal{R}} e^{-x^2-y^2}\ dA$ where $\mathcal{R}$ is bounded by $x = \sqrt{4-y^2}$ and $x = 0$.

If $x = 0$, then $y \in [-2, 2]$, hence our region is:

$$\mathcal{R} = \{(x,y) : -2 \le y \le 2, 0 \le x \le \sqrt{4-y^2}\} \quad \Rightarrow \quad \mathcal{R} = \{(r,\theta) : -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, 0 \le r \le 2\}$$

So our integral is

$$\iint_{\mathcal{R}} e^{-x^2-y^2}\ dA = \int_{-2}^{2}\int_0^{\sqrt{4-y^2}} e^{-x^2-y^2}\ dxdy$$

Converting to polar coordinates we have

$$\begin{aligned}
\int_{-\pi/2}^{\pi/2}\int_0^2 e^{-r^2}\ rdrd\theta &= \int_{-\pi/2}^{\pi/2} d\theta \int_0^2 e^{-r^2}\ rdr \\
&= \left(\theta\Big|_{-\pi/2}^{\pi/2}\right)\left(-\frac{e^{-r^2}}{2}\Big|_0^2\right) \\
&= -\frac{\pi}{2}(e^{-4} - e^0) = \frac{\pi}{2}(1 - e^{-4})
\end{aligned}$$

(5) **2.** Find the area of the surface $z = -x^2 - y^2 + 4$ bounded by the xy-plane.

Looking at the projection of the region on the xy-plane we have the equation $x^2 + y^2 \le 4$. Hence our region in polar coordinates is:

$$\mathcal{R} = \{(r,\theta) : 0 \le r \le 2, 0 \le \theta \le 2\pi\}$$

and so our integral is

$$\begin{aligned}
\iint_{\mathcal{R}} \sqrt{1 + (f_x)^2 + (f_y)^2}\ dA &= \iint_{\mathcal{R}} \sqrt{1 + 4x^2 + 4y^2}\ dA \\
&= \int_0^{2\pi}\int_0^2 \sqrt{1+4r^2} rdrd\theta \\
&= \frac{1}{8}\int_0^{2\pi} d\theta \int_0^2 8r(1+4r^2)^{1/2} dr \\
&= \frac{1}{12}\left(\theta\Big|_0^{2\pi}\right)\left((1+4r^2)^{3/2}\Big|_0^2\right) \\
&= \frac{\pi}{6}(17^{3/2} - 1)
\end{aligned}$$