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Math 2D: Quiz 3

(5) 1. Find the vector function that represents the curve of intersection of the two surfaces. The paraboloid $z = 4x^2 + y^2$, and the parabolic cylinder $y = x^2$. What type of curve is this?

Since the projection of the curve C on the xy-plane is the parabola $y = x^2$. Let $x = t$, then $y = t^2$. Since C also lieson the surface $z = 4x^2 + y^2$, we have $z = 4t^2 + t^4$. Hence the vector function is

$$r(t) = \langle t, t^2, 4t^2 + t^4 \rangle$$

and the curve looks like a bent parabola. (See Figure 1)

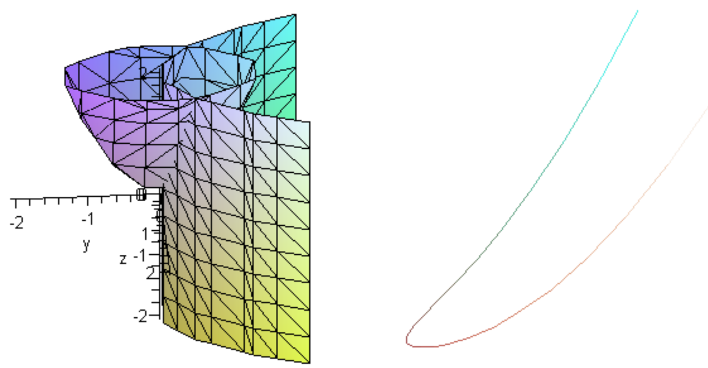

FIGURE 1

(5) 2. Find the length of the curve $r(t) = \langle 2\sin(t), 2\cos(t), \sqrt{5}t \rangle$, for $0 \le t \le 1$.

The length of the curve is given by $\int_0^1 \|r'(t)\| dt$.
So $r'(t) = \langle 2\cos(t), -2\sin(t), \sqrt{5} \rangle$, and $\|r'(t)\|$ is given by

$$\|r'(t)\| = \sqrt{4\cos^2(t) + 4\sin^2(t) + 5} = \sqrt{4(\cos^2(t) + \sin^2(t)) + 5} = \sqrt{9} = 3$$

Hence the arc length is

$$\int_0^1 \|r't\| dt = \int_0^1 3 dt = 3$$