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Math 147: Quiz 5

(10) **1.** Let $f(z)$ be entire holomorphic function on $\mathbb{C}$ such that $|f(z)| \leq |\cos(z)|$ for all $z \in \mathbb{C}$. Prove $f(z) = c\cos(z)$ for some constant c. *Hint: use Liouville's thoerem.*

Proof: Consider $g(z) = \dfrac{f(z)}{\cos(z)}$, then $|g(z)| \leq 1$, hence $g(z)$ is a bounded function. Define $\widehat{g}(z)$ as follows:

$$\widehat{g}(z) = \begin{cases} g(z) & \text{if } \cos(z) \neq 0 \\ \lim\limits_{z \to w} g(z) & \text{if } \cos(w) = 0 \end{cases}$$

Then $\widehat{g}(z)$ is a bounded entire function. Hence by Lioville's theorem it must, i.e. $\widehat{g}(z) = c$ for some $c \in \mathbb{C}$. It follows from the definition of $g(z)$ that $f(z) = c\cos(z)$ $\square$