Name:
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Math 147: Quiz 2

(5) **1.** Prove that the function $f : z \to |z|$ is continuous. *Hint:* Start with a Cauchy sequence $z_j \to z$.

Proof: Let $\{z_j\} \to z$ be a Cauchy sequence. By definition of a Cauchy sequence we have

$$\forall \epsilon > 0 \; \exists N \; s.t. \; |z_n - z_m| < \epsilon \; \forall n, m > N$$

Using the reverse triangle inequality we have

$$|f(z_n) - f(z_m)| = ||z_n| - |z_m|| \leq |z_n - z_m| < \epsilon,$$

for n and m large enough. So we have

$$\forall \eta > 0 \; \exists \delta (= \epsilon) \; s.t. \; |z_n - z_m| < \delta \quad \Rightarrow \quad |f(z_n) - f(z_m)| \leq \eta,$$

or $f(z) = |z|$ is continuous.

(5) **2.** Let $F(z)$ be a holomorphic function such that $F : \mathbb{C} \to \mathbb{R}$. Show that $F(z)$ is identically constant.

Let $F(z) = u(x, y) + \imath v(x, y)$, since $F : \mathbb{C} \to \mathbb{R}$, we must have for all $z \in \mathbb{C}$, $v(x, y) = 0$. Now consider the Cauchy-Riemann equations. Since $v(x, y) = 0$, we have $v_x = v_y = 0$, and so

$$\begin{cases} u_x = v_y = 0 \\ u_y = -v_x = 0 \end{cases}$$

Now these equations hold for all $z = x + \imath y \in \mathbb{C}$. Since $u_x(x, y) = u_y(x, y) = 0$, we conclude that $u(x, y) = \alpha \in \mathbb{R}$, i.e. $f(z)$ is identically constant. $\square$