Name:
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Math 147: Quiz 3

(5) **1.** Let γ be the unit circle traversed in the CLOCKWISE direction. Compute $\displaystyle\int_\gamma \frac{1}{z}\, dz$.

Solution: First notice that

$$\frac{1}{z} = \frac{\overline{z}}{|z|^2},$$

applying this with the definition of a contour integral we have,

$$\int_\gamma \frac{1}{z}\, dz = \int_a^b \frac{\overline{z}(\gamma(t))}{|z(\gamma(t))|^2}\gamma'(t)\, dt.$$

Now a parameterzation for $\gamma(t)$ is:

$$\gamma(t) = \cos(t) - \imath \sin(t), \qquad t \in [0, 2\pi].$$

With this parameterization notice that $|z(\gamma(t))|^2 = 1$, Plugging this in to the integral we have,

$$\begin{aligned} \int_0^{2\pi} \overline{\gamma(t)}\gamma'(t)\, dt &= \int_0^{2\pi} (\cos(t) + \imath\sin(t))(-\sin(t) - \imath\cos(t))dt \\ &= -\imath \int_0^{2\pi} dt = -2\pi\imath \;\square \end{aligned}$$

(5) **2.** Let $U \subset \mathbb{C}$, U open and let $f(z) \in C^1(U)$. Suppose that $\gamma : [0,1] \to U$ such that $\gamma \in C^1[0,1]$ and $\gamma(0) = \gamma(1)$. Prove that,

$$\int_\gamma f'(z)\, dz = 0.$$

Proof:

$$\begin{aligned} \int_\gamma f'(z)\, dz &= \int_0^1 f'(\gamma(t))\frac{d}{dt}\gamma(t)\, dt \\ &= \int_0^1 \frac{d}{dt} f(\gamma(t))dt = f(\gamma(1)) - f(\gamma(0)) = 0 \;\square \end{aligned}$$