Name:
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Math 147: Quiz 6

(5) **1.** Prove that the series $\sum_{j=0}^{\infty} \frac{1}{z^j}$ converges uniformly on the set $D = \{z : |z| > 5\}$

Proof: First notice that if $|z| > 5$, then $\frac{1}{|z|} < \frac{1}{5}$, and so

$$\left| \sum_{j=0}^{\infty} \frac{1}{z^j} \right| \leq \sum_{j=0}^{\infty} \frac{1}{|z|^j} \leq \sum_{j=0}^{\infty} \frac{1}{5^j} < \infty.$$

Therefore the series converges uniformly, on the set D, by the Weierstras M-test.

(5) **2.** Describe the set D such that the series $\sum_{j=0}^{n} \frac{j^2}{3^j} z^j$ converges

Solution: the radius of convergence is given by $\limsup_{j\to\infty} \left(\frac{3^j}{j^2}\right)^{1/j}$, computing this we see that $D = \{z : |z| < 3\}$

(5) **3.** Suppose that $f(z)$ is entire and that

$$\lim_{k\to\infty} \frac{\partial^k f(z)}{\partial z^k}$$

exists, uniformly on compact sets and that this limit is not identically zero. What can you say about the limit? *Hint: If $F(z)$ is the limit, then how is $F'(z)$ related to $F(z)$?*

Solution: Let $F(z)$ denote the limit, now notice that

$$\lim_{k\to\infty} \frac{\partial^k f(z)}{\partial z^k} = F(z) \text{ and } \lim_{k\to\infty} \frac{\partial^{k+1} f(z)}{\partial z^{k+1}} = F'(z) = F(z)$$

Solving this we see that $F(z) = Ae^(z)$ for some nonzero constant $A \in \mathbb{C}$.