Name:
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Math 147: Quiz 1

(5) **1.** Find all the roots of $f(z) = z^2 + z + 1$, and write them in polar form.

Solution: There are a couple different approaches to take, the easiest is to use the quadratic formula.

$$z^2 + z + 1 \qquad \Rightarrow \qquad z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\imath$$

Now $|z| = r = 1$ and $\theta = \tan^{-1}\left(\pm\sqrt{3}\right) = \dfrac{2\pi}{3}$, or $\dfrac{4\pi}{3}$. So the solutions in polar coordinates are

$$z = e^{\imath\left(\frac{2\pi}{3}+2k\pi\right)}, \quad e^{\imath\left(\frac{4\pi}{3}+2k\pi\right)} \text{ for } k \in \mathbb{Z}$$

(5) **2.** Let $\theta, a \in \mathbb{R}$, such that $a > 0$, prove that: $(\cos(\theta) + \imath\sin(\theta))^a = \cos(a\theta) + \imath\sin(a\theta)$.

Proof: Using Euler's formula $e^{\imath\theta} = \cos(\theta) + \imath\sin(\theta)$, we have

$$\begin{aligned}(\cos(\theta) + \imath\sin(\theta))^a &= \left(e^{\imath\theta}\right)^a \\ &= e^{\imath a\theta} \\ &= \cos(a\theta) + \imath\sin(a\theta) \ \square\end{aligned}$$