**1.** Let $f : \mathbb{R} \to \mathbb{R}^n$ be differentiable such that $\|f(t)\|$ is always constant. Prove that $f(t) \cdot f'(t) = 0$.

**2.** Suppose $F$ is a continuous function from $B(0,1) \subset \mathbb{R}^n$ into itself, and that $\|F(x)\| < \|x\|$ for any $x \in B(0,1) \setminus \{0\}$. Fix $x_0 \in B(0,1) \setminus \{0\}$, and let $x_{k+1} = F(x_k)$ for $k \in \mathbb{N}$. Prove that limit $x_k$ exists, and equals 0.

**3.** Let $S$ be convex and $F : S \to \mathbb{R}^n$ s.t $F \in C^3(S)$. Now suppose that $c \in S$ is such that all first and second order partial derivatives, but not all third order partial derivatives, vanish at $c$.

a) Compute the third order Talor expansion of $F(x)$.
b) How do $f(x) - f(c)$, from the third order Talor expansion, and $D^3_{x-c} f(c)$ differ?
$D^3_{x-c} f(c)$ is the third order directional derivative in the direction of $x - c$ at the value $c$.

**4.** Let $S$ be open connected in $\mathbb{R}^n$ and $F : S \to \mathbb{R}$ s.t. $F \in C^1(S)$ if $Df(c) = 0$ for all $c \in S$ show that $F(x)$ is constant on $S$.