1. Let f and g be real valued, suppose $\nabla f(x)$, $\nabla g(x)$ exist at a point $x \in \mathbb{R}^n$. Show that the product $h(x) = f(x)g(x)$ has a gradient vector at x defined by

$$\nabla h(x) = f(x)\nabla g(x) + g(x)\nabla f(x)$$

2. Let f and g be real valued functions on $\mathbb{R}$ such that $f, g \in C^2(\mathbb{R}^2)$. Define

$$F(x, y) = f[x + g(y)] \text{ for each } (x, y) \in \mathbb{R}^2$$

Show that $F_x F_{xy} = F_y F_{xx}$.

3. Let $f : \mathbb{R}^n_+ \to \mathbb{R}$ be continuous and differentiable. Let $\lambda \in \mathbb{R}$ and suppose that for some $p \in \mathbb{N}$ we have $f(\lambda x) = \lambda^p f(x)$ for all $\lambda \in (0, \infty)$ and all $x \in \mathbb{R}^n_+$. Show that:

$$x \cdot \nabla f(x) = pf(x)$$

Hint: Define an auxillary function $g(\lambda)$ and compute $g'(1)$