Exercise 2.2.13: Let $f : X \to Y$, show that

$$a)\ ff^{-1}(T) \subset T \qquad\qquad b)\ S \subset f^{-1}(T)$$

where $S \subset X, T \subset Y$. Note, I changed the notation of the problem.

Solution: a) it is equivalent to show that if $S = f^{-1}(T)$, then $f(S) \subset T$. Let $b \in f(S)$, this implies there exists a $A \subset T$, such that $b = f(a)$ for all $a \in A$, hence we have

$$f^{-1}(b) = A \subset S \quad \Rightarrow \quad f(f^{-1}(b)) = f(A) = b \in T \quad \Rightarrow \quad ff^{-1}(T) \subset T$$

For b) it is equivalent to show that if $T = f(S)$, then $S \subset f^{-1}(T)$. Let $a \in S$, then there exists some $b \in T$ such that $f(a) = b$, then we have $a \in f^{-1}(b) \subset f^{-1}(T)$, and hence $S \subset f^{-1}(T)$.

Exercise 2.2.14: $f(x) = F(x, c)$ is continuous for $x \in [-1, 1]$ if $|c| \leq 1$, otherwise F(x,c) does not have to be continuous at all. To see this consider the following function: Let $S = \{(x, y) : |x| \leq 1, |y| \leq 1\}$

$$F(x,y) = \begin{cases} x+y & S \\ -1 & \mathbb{Q} \times \mathbb{Q} - S \\ -2 & \mathbb{Q}^c \times \mathbb{Q} - S \\ 2 & \mathbb{Q} \times \mathbb{Q}^c - S \\ 1 & \mathbb{Q}^c \times \mathbb{Q}^c - S \end{cases}$$

If $c = 2$, for example, then $F(x, 2)$ is not continuous anywhere. To show that $F(x, c)$ is continuous for $|c| \leq 1$ use the definition of continuity and the iterated limits.(I did something similiar in discussion)