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Math 121B: Quiz 2

(5) **1.** Let be a T linear operator on a finite dimensional vector space V with $\dim(V) = n$.
Suppose that T has n distinct eigenvalues, prove that T is diagonalizable.

Proof: First note there are multiple solutions to problem. I will give you the easiest one in my opinion. Let $\{\lambda_i\}$ be the set of eigenvalues. If the all the eigenvalues of T are distinct, then $\lambda_i \neq \lambda_j$ for $i \neq j$. Now the set of eigenvalues are the roots of the characteristic equations for T, so by assupmtion the characteristic equation splits [1],

$$c_T(\lambda) = \prod_{i=1}^{n} (\lambda - \lambda_i)^{n_i}.$$

Also by assumption $n_i = 1$, for all i, this implies that the algebraic multiplicity for each eigenvalue is 1. Now for the geometric multiplicity is bounded by $1 \leq \dim(\mathcal{E}_{\lambda_i}) \leq n_i$. So we must have $\dim(\mathcal{E}_{\lambda_i}) = n_i = 1$. In otherwords the algebraic multiplicity equals the geometric multiplicty. [2]
T is diagonalizable if and only if conditions [1] and [2] are satisfied. $\square$

(5) **2.** Show that if λ_1, λ_2 are distint eigenvalues for a linear operator T, then $\mathcal{E}_{\lambda_1} \cap \mathcal{E}_{\lambda_2} = \{0\}$

Solution: Want to do this by contradiction, so suppose that $0 \neq v \in \mathcal{E}_{\lambda_1} \cap \mathcal{E}_{\lambda_2}$. By definition of eigenvalue and eigenvector we have

$$Tv = \lambda_1 v \text{ and } Tv = \lambda_2 v \quad \Rightarrow \quad \lambda_1 v = \lambda_2 v \quad \Leftrightarrow \quad \lambda_1 = \lambda_2 \text{ or } v = 0$$

In either case we have a contradiction. Therefore $\mathcal{E}_{\lambda_1} \cap \mathcal{E}_{\lambda_2} = \{0\}$ $\square$