Name:
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Math 121B quiz 1

1. Prove that 2 similar matricies share the same multiset of eigenvalues.
Hint: use the definition of similar, i.e. for $A, B, \in \mathcal{M}_n(\mathbb{F})$, $\exists P \in \mathcal{M}_n(\mathbb{F})$ such that $A = PBP^{-1}$.

Proof: Let λ be an eigenvalue for A and v be a correspoding eigenvector. Then we have,

$$\begin{aligned} Av &= PBP^{-1}v = \lambda v \text{ and so} \\ P^{-1}PBP^{-1}v &= P^{-1}\lambda v \end{aligned}$$

Let $w = P^{-1}v$, then the above implies that $Bw = \lambda w$. Since λ is an arbitrary eigenvaule for A, we can conculde that 2 similar matricies share the same multiset of eigenvalues. $\square$

2. Prove or disprove that 2 similar matricies share the same set of eigenvectors.

Proof: Let λ be an eigenvalue for A and v, from problem one we have there exists a w such that

$$Av = \lambda v \qquad Bw = \lambda w$$

Now also from the work done in problem 1, $w = Qv$ for some matrix Q. Suppose that the multiplicity of λ from the characteristic equation is one. Then,

$$w - v = 0 \quad \Rightarrow \quad Qv - v = 0 \quad \Rightarrow \quad (Q - I)v = 0$$

This implies that either $v = 0$, which cannot happen, or $Q = I$. Therefore 2 similar matricies do not have to share the same set of eigenvectors. $\square$