Name:
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Math 121B: Quiz 4

(3) **1.** State the definition and give an example of an inner product space

Solution: An inner product space is a vector space V, equipped with and inner product $\langle\cdot,\cdot\rangle$. Where $\langle\cdot,\cdot\rangle$ is:
conjugate bilinear
conjugate symmetric
$\langle x, x\rangle > 0$ for all $0 \neq x \in V$
$\mathbb{R}^n$ equipped with the dot product $\langle v, w\rangle = \sum_{i=1}^n v_i w_i$ defines and inner product space.

(5) **2a.** Let $x, y \in (V, \langle\cdot,\cdot\rangle)$ prove the triangle inequality; $\|x+y\| \leq \|x\| + \|y\|$.
Hint: you can use the Cauchy-Schwarz inequality in the proof.

Proof: Let $x, y \in V$

$$\begin{array}{rcl} \|x+y\|^2 & = & \langle x+y, x+y\rangle \\ & = & \langle x,x\rangle + \langle y,x\rangle + \langle x,y\rangle + \langle y,y\rangle \\ & \leq & \|x\|^2 + 2|\langle x,y\rangle| + \|y\|^2 \\ \text{Cauchy-Schwarz} & \leq & \|x\|^2 + 2\|x\|\cdot\|y\| + \|y\|^2 \\ & = & (\|x\| + \|y\|)^2 \end{array}$$

Taking squareroots of the first and last line shows the inequality.

(2) **2b.** When is $\|x+y\|^2 = \|x\|^2 + \|y\|^2$?

If $x \neq cy$, for some $c \in \mathbb{F}$ then the triangle inequality is equal when $\mathcal{R}(\langle x, y\rangle) = 0$, i.e x and y are orthogonal.