Name:
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Math 121B: Quiz 5

(5) **1.** Prove that every orthonormal set in a finite-dimensional inner product space is linearly independent

Proof: Let $S = \{v_i\}_{i=1}^k$ be a orthonormal set in $(V, \langle\cdot,\cdot\rangle)$, and let $y \in$span$\{v_i\}$, so

$$y = \sum_{i=1}^k a_i v_i$$

for some scalars a_i. We want to show that if $y = 0$, then $a_i = 0$ for $i = 1..k$. Since S is an orthonormal set the coeffients $a_i = \langle y, v_i\rangle$. So if $y = 0$ we have

$$0 = y = \sum_{i=1}^k \langle y, v_i\rangle v_i = \sum_{i=1}^k \langle 0, v_i\rangle v_i$$

and so $y = 0$ implies $a_i = \langle 0, v_i\rangle = 0$, hence the set S is linearly independent. $\square$

(5) **2.** Let $\{v_1, \ldots v_k\}$ be an orthogonal set in $(V, \langle\cdot,\cdot\rangle)$ and let $a_1, \ldots, a_k$ be scalars. Prove that

$$\left\| \sum_{i=1}^k a_i v_i \right\|^2 = \sum_{i=1}^k |a_i| \|v_i\|^2.$$

Hint: $\|x + y\|^2 = \langle x + y, x + y\rangle$

Proof:

$$\begin{aligned} \left\| \sum_{i=1}^k a_i v_i \right\|^2 &= \left\langle \sum_{i=1}^k a_i v_i, \sum_{j=1}^k a_j v_j \right\rangle \\ \text{by conjugate bilinearity} &= \sum_{i=1}^k \sum_{j=1}^k a_i \overline{a}_j \langle v_i, v_j\rangle \\ \text{by orthonormality} &= \sum_{i=1}^k |a_i| \langle v_i, v_i\rangle \\ &= \sum_{i=1}^k |a_i| \|v_i, v_i\|^2 \quad \square \end{aligned}$$