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Math 121B: Quiz 3

(5) **1.** Let T be a linear operator on a finite-dimensional vector space V with dimension n. Prove that there exists a polynomial $g(t)$ of degree n, such that $g(T)v = 0$ for all $v \in V$.

Proof: Let $c_T(\lambda)$ be the characteristic equation for T. Now $\deg(c_T(\lambda)) = n = \dim(V)$. By the Cayley-Hamilton theorem we have that $c_T(T) = T_0$, where T_0 is the zero transformation. Thus we have for any $v \in V$

$$c_T(T)v = T_0 v = 0 \in V \ \square$$

(5) **2.** Let V be a finite-dimensional vector space and $T \in \mathcal{L}(V)$. Let

$$c_T(t) = (-1)^n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0$$

be the characteristic polynomial of T. Prove that T is invertible if and only if $a_0 \neq 0$.

Proof: Since $c_T(\lambda) = \det(T - tI)$, by evaluating both sides at 0, we get have $c_T(0) = \det(T) = a_0$. So, T is invertible if and only if $\det(T) \neq 0$ if and only if $a_0 \neq 0$ $\square$.