Math 4B:
Section 2, Jan 11th. Time:

Solve the following IVP. Sketch their solutions and determine approximately the interval of existence for each.

\[ y' = (1 - 2x)y^2 \]
\[ y(0) = -1/6 \]  
\[ y = \frac{1}{x^2 - x - 6} = \frac{1}{(x-3)(x+2)} \]  
\[ y' = (3x^2 - e^x)/(2y - 5) \]
\[ y(0) = 1 \]

\[ (2y - 5)y' = 3x^2 - e^x \]

Integrate

\[ y^2 - 5y = x^3 - e^x + c \]

\[ 1 - 5 = 0 - 1 + c \]

\[ c = -3 \]

Interval of Existence

\[ z \in (-1.4, 4.6) \]

Plot with computer

=>

Interval of Existence

\[ z \in (-1.4, 4.6) \]
Solve the following IVP.

\[ y' + 2y = te^{-2t} \]
\[ y(1) = 0 \]  \hspace{1cm} (5)
\hspace{1cm} (6)

\[ M(t) = e^{2t} \]
\[ e^{2t} y' + 2e^{2t} y = t \]
\[ [e^{2t} y]' = t \]
\[ e^{2t} y = \frac{t^2}{2} + c \]

Use \( t, c \):
\[ c = -\frac{1}{2} \]

\[ M(t) = e^{t + 1 + t} = te^{2t} \]
\[ t e^{2t} y' + (t + 1)e^{2t} y = te^{2t} \]
\[ [te^{2t} y]' = te^{2t} \]
\[ te^{2t} y = (t - 1)e^{2t} + c \]

Use \( t, c \):
\[ c = \frac{2}{t} \]  \hspace{1cm} (7)
\[ y(\ln 2) = 1, \quad t > 0 \]  \hspace{1cm} (8)

\[ y' + (1 + \frac{1}{t}) y = 1 \]
\[ y = 1 - \frac{1}{t} + \frac{2}{te^t} \]