PDE'S FINITE ELEMENT METHOD
HOMEWORK 5

1. Consider the linear system:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 19 \\ 19 \\ -3 \\ -12 \end{bmatrix}$$

   (a) Calculate the exact solution using Gaussian Elimination.
   (b) Calculate the first 10 iterates ($\mathbf{u}_k$, $k = 1 \dots 10$) obtained with Jacobi, Gauss-Seidel and S.O.R. with $\omega = 1.1, 1.2, \dots 1.9$, taking $\mathbf{u}_0 = \mathbf{0}$ as your initial guess. Which method gives you the best performance?

2. Let K be the reference triangle with vertices $\mathbf{a}^1 = (0, 0)$, $\mathbf{a}^2 = (1, 0)$, and $\mathbf{a}^3 = (0, 1)$ and $g(\mathbf{x})$ be an integrable function in K. Prove that the quadrature

$$Q_v[g] = \frac{1}{6} \sum_{i=1}^{3} g(\mathbf{a}^i)$$

   gives the exact value of the integral

$$\int_K g(\mathbf{x}) d\mathbf{x}$$

   when g is a polynomial of degree less or equal than 1, i.e.

$$\int_K g(\mathbf{x}) d\mathbf{x} - Q_v[g] = 0$$

   for $g \in P_1$.