

# PDE'S FINITE ELEMENT METHOD

## HOMEWORK 3

Consider the Poisson equation with homogeneous Dirichlet boundary conditions:

$$\begin{aligned} -\Delta u &= f && \text{in } \Omega, \\ u &= 0 && \text{on } \Gamma, \end{aligned}$$

where  $\Gamma$  is the boundary of  $\Omega$ .

1. Compute the stiffness matrix when  $\Omega$  is the triangle with vertices  $(0, 0)$ ,  $(1, 0)$ , and  $(1, 1)$ , using a uniform triangulation and piecewise linear ( $P_1$ ) elements.
2. Compute the stiffness matrix for  $\Omega$  equal to the unit square using a uniform triangulation and piecewise linear ( $P_1$ ) elements.
3. a) Show that the element stiffness matrix for linear polynomials on a triangle  $K$  with vertices  $(0, 0)$ ,  $(h, 0)$ , and  $(0, h)$ , numbered 1, 2, and 3, is given by

$$A^K = \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1/2 & 0 \\ -1/2 & 0 & 1/2 \end{bmatrix}$$

- b) Use this to verify your answer in Problem 2.
- c) Compute the element stiffness matrix for a triangle  $K$  with nodes  $a^1$ ,  $a^2$ , and  $a^3$ .