

PDE'S FINITE ELEMENT METHOD HOMEWORK 1

1. Using a uniform mesh with mesh size $h = 0.25$ compute the Galerkin piecewise linear finite element approximation of $-u'' = x$, $u(0) = u(1) = 0$ by hand.
2. An $M \times M$ matrix A is positive definite if $\eta^T A \eta > 0$ for all M -vectors $\eta \neq 0$. Show that the matrix in the Galerkin approximation for $-u'' = f$, $u(0) = u(1) = 0$ is positive definite. (hint: use that $\eta^T A \eta = \int_0^1 v^2 dx$ where $v = \sum_i \eta_i \phi_i(x)$)
3. Compute the stiffness matrix and load vector for the Galerkin piecewise linear FEM on a uniform mesh for $-(au')' = f$ with $a(x) = 1 + x$ and $f(x) = \sin(x)$, and the Dirichlet boundary conditions $u(0) = u(1) = 0$.
4. Compute the Galerkin piecewise linear FEM on a uniform mesh for $a = f = 1$ and $a(1)u'(1) = 1$, $u(0) = 0$.
5. Compute the matrix A and the vector b for problem 3 but using the trapezoidal rule to evaluate the integrals.