

Practice Final Math 104B

Name:

Instructions: Write your name clearly. Do all the problems. Show all your work.

1. True (T) or False (F). Suppose A is an $n \times n$ positive definite matrix:
 - a) () The search directions for the conjugate gradient (CG) method are always the residuals.
 - b) () The CG method will converge to the exact solution of $A\mathbf{x} = \mathbf{b}$ in at most n^2 iterations.
 - c) () Two vectors \mathbf{u} and \mathbf{v} are said to be conjugate with respect to A if and only if $\mathbf{u}^T A \mathbf{v} > 0$.
 - d) () The most expensive part in a CG iteration is computing the product of A and a vector.
 - e) () For A sparse, the CG method generally beats Jacobi, GS, and S.O.R.
2. The concentration c of a radioactive material decays according to the law $c(t) = be^{-at}$ where t represents time in seconds, $a = 0.1 \text{ sec}^{-1}$, and b is the initial concentration.
 - a) Using the Least Squares method and the data table (Table 1) below find b .
 - b) Find

| t_i (sec) | C_i |
|-------------|-------|
| 1 | 0.91 |
| 2 | 0.80 |
| 3 | 0.76 |
| 4 | 0.65 |

Table 1:

- the error in the least squares approximation.
3. Given a collection of data points $\{(x_i, y_i)\}_{i=1}^m$ find the best least squares approximation of the form $y = ax^2 + bx^3$.
 4. (a) Given a collection of data points $\{(x_i, y_i)\}_{i=1}^m$ find the best least squares approximation of the form $y = ax + bx^2$.
 - (b) Use this approximation to fit the data in Table 2.
 - (c) Find the error in the least squares approximation.
 5. a) Obtain the first 4 Legendre polynomials. b) Find the least squares polynomial approximations of degrees 1, 2, and 3 for the function $f(x) = e^x$ on $[-1, 1]$. c) What is the polynomial least squares approximation of degree 4 for $f(x) = x^3$ on $[-1, 1]$? Explain

| x_i | y_i |
|-------|-------|
| 1 | 3.1 |
| 2 | 9.8 |
| 3 | 21.2 |
| 4 | 36.1 |

Table 2:

6. Find the general continuous least squares trigonometric polynomial $S_n(x)$ for square pulse:

$$f(x) = \begin{cases} -1 & \text{if } -\pi \leq x \leq 0, \\ 1 & \text{if } 0 < x \leq \pi. \end{cases}$$

7. a) Determine the interpolating trigonometric polynomial $S_4(x)$ of degree 4 for $f(x) = e^{\sin(x)}$ on $[-\pi, \pi]$. b) Find $\max_i |S_4'(x_i) - f'(x_i)|$.

8. Circle the right answer(s) (no partial credit). The FFT is based on:

- a) the interpolation accuracy of the least squares polynomials.
- b) grouping complex Fourier coefficients in pairs.
- c) the identity $e^{in\pi} = (-1)^n$.
- d) none of the above.

9. Write a summary of what you learned in this course.